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WASHINGTON OBSERVATIONS FOR 1870.—APPENDIX III.

ON THE RIGHT ASCENSIONS

OF THE

EQUATORIAL FUNDAMENTAL STARS

AND THE CORRECTIONS NECESSARY TO REDUCE THE

RIGHT ASCENSIONS OF DIFFERENT CATALOGUES

TO A

MEAN HOMOGENEOUS SYSTEM.

BY

SIMON NEWCOMB,
PROFESSOR OF MATHEMATICS, UNITED STATES NAVY.

PREPARED AT THE U. S. NAVAL OBSERVATORY
BY ORDER OF
REAR-ADMIRAL B. F. SANDS, U. S. N.,
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ON THE RIGHT ASCENSIONS

OF THE

EQUATORIAL FUNDAMENTAL STARS

AND

THE CORRECTIONS NECESSARY TO REDUCE THE RIGHT ASCENSIONS OF DIFFERENT STAR-CATALOGUES TO A MEAN HOMOGENEOUS SYSTEM.

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§ 1.

INTRODUCTION.

Within the past few years very important additions have been made to the material at our disposal for an accurate determination of the positions and proper motions of the fundamental stars. These comprise Dr. Auwers' reduction of Bradley's observations, the Pulkowa catalogue of 1845, and the continuation of the Greenwich and Washington observations. It is not too much to say that this additional material alone would suffice to furnish results far more accurate than any that could be obtained from the older data.

The object of the present work is to do for the right ascensions of the equatorial and zodiacal stars, on which the reductions of lunar and planetary observations depend, what has been done by Dr. Auwers for the declinations, namely, to furnish the data necessary to reduce the principal original catalogues of stars to a homogeneous system by freeing them of their systematic differences. I think this can best be done by comparing the catalogue-positions of the fundamental stars with a uniform set of standard positions as free as possible from systematic error, and to consider any systematic discordance between the standard and the catalogue position as due to error of the latter, and correct it accordingly. For the reason already mentioned, it will not be admissible to take any existing catalogue as the standard. The first step of our work is, therefore, the preparation of a system of right ascensions of fundamental stars which shall be as free as possible from the suspicion of systematic error.

In the formation of such a catalogue I have been guided by certain considerations respecting the probable presence of systematic errors in the reduced right ascensions of many modern catalogues. It has long been the custom, in the reduction of right-ascension observations, to deduce a correction of the clock from adopted right ascensions of stars, and to consider the mean correction thus deduced as applicable to all the stars observed, the clock-stars included. On this system it is clear that if the assumed positions of all the clock-stars used on any one night are affected by a common error, that same error will be carried into the concluded positions. Thus any periodic error depending on the right ascension will tend to perpetuate itself. In a former paper I showed that if the clock-stars observed on each night are distributed uniformly over a period of six hours, and if the assumed positions are affected with the error—

$$a \sin \text{R. A.} + b \cos \text{R. A.} + a' \sin 2 \text{ R. A.} + b' \cos 2 \text{ R. A.}$$

the concluded positions will be affected with the error—

$$\frac{8}{\pi^2} (a \sin \text{R. A.} + b \cos \text{R. A.}) + \frac{4}{\pi^2} (a' \sin 2 \text{ R. A.} + b' \cos 2 \text{ R. A.})$$

Hence the error of single period will be cut down very slowly by successive revisions of the catalogue, each revision reducing it by less than one-fifth. On the other hand, the error of double period will rapidly disappear.

The introduction of these periodic errors is very easy, even where the greatest care is taken in the reductions, owing to the diurnal variation of the conditions to which the instrument, the clock, and the observer are subjected. If the pointing of the instrument, the rate of the clock, or the personal equation of the observer vary during the twenty-four hours, the right ascensions deduced will be affected by a corresponding error. It is true that if the variation continues uniform during an entire year, and if the observations are uniformly distributed throughout the year, this error will be eliminated from a year's work. But neither of these conditions is completely fulfilled, the diurnal change of temperature being greater in summer than in winter, while the annual period in the length of the night and the degree of cloudiness interferes with the continuity of the system of observation.

§ 2.

DETERMINATION OF A SYSTEM OF CORRECTIONS TO THE RELATIVE RIGHT ASCENSIONS OF BESSEL'S TABULÆ REGIOMONTANÆ, PRESUMED TO BE FREE FROM PERIODIC ERRORS.

For the reasons already mentioned, it seems best to deduce the periodic error of the standard catalogue solely from determinations which are entirely independent of any previous hypothesis respecting the right ascensions of clock-stars, and from those in which there is reason to believe that the periodic error has been entirely eliminated. I have selected the following twelve original catalogues as certainly or probably fulfilling one of these conditions:

1. *Auwers' Bradley*, 1755.—Dr. Auwers has very kindly placed his definitive results for relative right ascensions, which are as yet unpublished, at my disposal.

2. *Piazzi*, 1800.—"Praecipuarum stellarum inerrantium positiones mediæ ineunte saeculo XIX. Panormi, 1814."

3. *Maskelyne*, 1805.—Catalogue found in Maskelyne's Astronomical Observations made at the Royal Observatory, Greenwich, from 1799 to 1810, p. 112 of part 2.

4. *Auwers' Cacciatore*.—"Reduction der Beobachtungen der Fundamentalsterne am Passageninstrument der Sternwarte zu Palermo in den Jahren 1803 bis 1805, und Bestimmung der mittleren Rectascensionen für 1805. Von Arthur Auwers. Publication der astronomischen Gesellschaft, V. Leipzig, 1866."

5. *Bessel* I.—"Bestimmung der geraden Aufsteigungen der 36 Maskelyneschen Fundamental-Sterne für 1815, auf Königsberger Beobachtungen gegründet." Berlin Memoirs for 1818–'19, p. 19 of part 2.

6. *Bessel* II.—"Neue Untersuchungen über die geraden Aufsteigungen der 36 Fundamental-Sterne." Berlin Memoirs for 1825, p. 23 of part 2.

7. *Struve.*—Catalogue found in "Positiones mediæ," p. xxxxi.

8. *Brinkley.*—Catalogue found in the Astronomische Nachrichten, Band IV, S. 102.

9. *Argelander. Corrections given by Bessel in Astronomische Nachrichten, No.* 515.—These differ from the positions in Argelander's "DLX. Stellarum fixarum positiones mediæ" only in the addition of the third decimal of seconds.

10. *Pulkowa,* 1845.—"Observations de Poulkova, publiées par Otto Struve, Directeur de l'observatoire central Nicolas. Saint-Pétersbourg, 1869." Vol. I, p. 120.

11. *Greenwich,* 1864.—"New seven-year catalogue of 2760 stars deduced from observations extending from 1861 to 1867 at the Royal Observatory, Greenwich, and reduced to the epoch 1864." (Appendix II to the Greenwich observations for 1868.)

12. *Washington,* 1870.—"Positions of fundamental stars deduced from observations made at the United States Naval Observatory between the years 1862 and 1867." (Washington observations for 1867, Appendix III.)

On this selection I remark that I find no positive evidence that the catalogues of Piazzi and Brinkley are really independent of any previous determination, and that I have judged them to be so because the modern system of reduction had not then come into general use. Argelander, Greenwich, and Washington are not rigorously independent, but they are included, because, in the case of each, there is reason to believe that the systematic errors have been more or less completely eliminated. The following are the reasons in the case of each catalogue:

An examination of Argelander's observations shows that they were generally widely distributed throughout the twenty-four hours, his habit being to observe both morning and evening. From such a day's work the error of single period will be almost entirely eliminated.

The Memoirs of the Royal Astronomical Society, vol. 34, contain a paper by Mr. Stone, in which the stars γ Pegasi, Pollux, Spica, and α Aquilæ are compared, with the express purpose of detecting any inequality depending on the right ascension. The result is that the relative positions of these stars, as given in the seven-year catalogue for 1860, require the following corrections:

			s.		s.
γ Pegasi	—	Pollux,	−.001	or	−.014
Pollux	—	Spica,	+.029	or	+.015
Spica	—	α Aquilæ,	+.008	or	−.005
α Aquilæ	—	γ Pegasi,	+.018	or	+.004

The first column shows the differences independently deduced; the second their values when each is diminished by a constant so as to make their sum zero.

The smallness of this result indicates that the periodic error, if it exists at all, must be very small, and it ought to be smaller still in the catalogue of 1864, which includes still another revision.

In the case of Washington a special effort was made to determine the periodic error of Dr. Gould's right ascensions; which was of the form—

$$a \sin \mathrm{R.A.} + b \cos \mathrm{R.A.}$$

the details of which are given in the catalogue cited. The determination was entirely satisfactory except in one point—a large diurnal inequality showed itself during the second year. This inequality had to be determined on the supposition that it remained constant throughout the year, an hypothesis which, though probable, was not certain.

Having selected our data, the next question will be the mode of combining. It was my original intention to compare each catalogue with Dr. Gould's standard, and thus obtain a correction to the latter of the form—

$$a \sin \alpha + b \cos \alpha + c$$

A series of values of a, b, and c being thus found for the epochs of the various catalogues, the true values of these quantities should increase uniformly with the time. We should then determine a_o, a', b_o, b', c_o, c', from the equations—

$$\begin{aligned} a_o + a't &= a \\ b_o + b't &= b \\ c_o + c't &= c \end{aligned}$$

of which we should have as many sets as there were catalogues compared. Having found the most probable values of a_o, a', etc., the residuals left after substituting them in the equations given by each catalogue would show the systematic error of that catalogue.

But it has been deemed best to begin by determining each star independently of any catalogue whatever, and of any assumed proper motion. This is done most easily and elegantly by assigning the same weight to all the stars in any one catalogue; this weight being determined not by the general accuracy of the catalogue, but solely by its probable freedom from systematic errors. It will then be unnecessary to apply any reduction on account of equinox in this preliminary determination, because, if all the right ascensions of any one catalogue be changed by a constant, that change will affect all the concluded positions by the same amount, and therefore will not change the relative positions at all.

From the nature of the case, it is clearly impossible to make any numerical calculation of the relative weights to be assigned to the different determinations. This is necessarily a matter of individual judgment. Without going into the consideration of each particular case, I may remark that the following weights assigned were founded on considerations respecting the probable freedom of the instruments from sources of systematic error and the care with which systematic error was avoided in the observations and reductions, probable accidental errors not being taken into account, and that they were decided on before any comparison of results. They have, therefore, no necessary analogy with the weights to be assigned in a definitive discussion, after systematic error has been allowed for.

Weights assigned to observations.

No.	Catalogue.	Year.	Weight.
1	Auwers' Bradley . .	1755	3
2	Piazzi	1800	1
3	Maskelyne	1805	1
4	Auwers' Cacciotore .	1805	1
5	Bessel I	1815	1
6	Bessel II	1823	3
7	Struve	1824	3
8	Brinkley	1824	1
9	Argelander	1828	2
10	Pulkowa	1845	6
11	Greenwich	1864	2
12	Washington.	1870	3

The dates here given to the catalogues are generally supposed to be about the means of the times of observation on which they depend. The principal exceptions are Brinkley and Washington. The mean date of the former could not be inferred with certainty, but must be earlier than 1824. The date assigned for the latter is, through inadvertence, that to which the catalogue was reduced, which is five years later than the actual mean of the times of observation. The reduction to the epoch was, however, made with Dr. Gould's proper motions, which are so near the truth that few or none of the results are altered the thousandth of a second by this error. I have, therefore, not corrected it.

The results are all exhibited in the form of corrections to the mean positions and annual variations of the Tabulæ Regiomontanæ for the sake of convenience, and the principal steps of the computation are exhibited in the following tables. The first column in each table shows the correction to the right ascension of the Tabulæ Regiomontanæ given by the catalogue. Following one star through the several catalogues, we have a series of corrections applicable at the various epochs. The true correction being of the form $x + y\,t$, each catalogue result gives an equation of condition between x and y. As an epoch near the mean leads to the most convenient computations, I have chosen the year 1820 as the origin of time. Let us now represent by—

$$c_1,\ c_2,\ c_3,\quad - \quad - \quad - \quad -\quad c_{12}$$

the corrections given by the several catalogues, the subscript numbers being the same as those of the catalogues to which they correspond in the preceding list. Taking the century as the unit of time, the equations of condition will then be—

$$\begin{gathered} x - 0.65\,y = c_1 \\ x - 0.20\,y = c_2 \\ x - 0.15\,y = c_3 \\ - \quad - \quad - \quad - \quad - \\ - \quad - \quad - \quad - \quad - \\ \text{etc., etc., etc.} \end{gathered}$$

The solution of these equations by least squares, using the weights in the preceding list, will give the following values of x and y:

$$\begin{array}{rl@{\qquad}rl}
x = & +0.17\,c_1 & y = & -0.78\,c_1 \\
 & +0.04\,c_2 & & -0.10\,c_2 \\
 & +0.04\,c_3 & & -0.08\,c_3 \\
 & +0.04\,c_4 & & -0.08\,c_4 \\
 & +0.04\,c_5 & & -0.03\,c_5 \\
 & +0.11\,c_6 & & -0.04\,c_6 \\
 & +0.11\,c_7 & & -0.03\,c_7 \\
 & +0.03\,c_8 & & -0.01\,c_8 \\
 & +0.08\,c_9 & & +0.01\,c_9 \\
 & +0.19\,c_{10} & & +0.40\,c_{10} \\
 & +0.06\,c_{11} & & +0.27\,c_{11} \\
 & +0.09\,c_{12} & & +0.47\,c_{12}
\end{array}$$

No account is here taken of the discordant equinoxes of the various catalogues, because, all the stars in the same catalogue having the same weight, a change of equinox will not change the relative positions of the stars.

In the selection of stars, attention has been at first confined to the Maskelyne fundamental stars between the limits of declination + 40° and — 30°. Several stars have to be omitted, namely, Procyon and Sirius, on account of the irregularity of their proper motions which has been so exhaustively investigated by Auwers that no further discussion of their positions is yet necessary; Castor, on account of its duplicity, and the confusion of the observations upon it, which render it unfit for use as a standard star; β Virginis, α' Libræ, and α' Capricorni, because they have of late dropped out of use.

Under each catalogue, in the following table, we have, in column c, the correction to the Tabulæ Regiomontanæ given by the catalogue, from which the values of x and y are deduced by the preceding formulæ.

The value of the correction $x + ty$ is next computed for the epoch of each catalogue, and subtracted from the correction given by observation. The excess of the catalogue result thus obtained is given in the second column of the table. To correct the residuals for discordance of equinoxes, the mean value of the numbers in this column is taken, and subtracted from the individual numbers. The remainders represent the discordances between the catalogue and the correction as finally deduced.

Corrections to right ascensions given by standard catalogues.

Star.	Bradley, 1755.			Piazzi, 1800.		
	c_1	c'	c_1''	c_2	c_2'	c_2''
γ Pegasi	+ 65	0	− 31	− 54	− 65	+ 18
α Arietis	− 56	+ 40	+ 9	− 168	− 144	− 61
α Ceti.	+ 38	+ 64	+ 33	− 76	− 86	− 3
α Tauri	− 12	+ 51	+ 20	− 194	− 172	− 89
β Orionis. . . .	+ 40	+ 33	+ 2	− 54	− 36	+ 47
β Tauri	+ 152	+ 31	0	− 53	− 100	− 17
α Orionis. . . .	+ 30	+ 22	− 9	− 7	− 13	+ 70
β Geminorum . .	− 102	+ 41	+ 10	− 173	− 133	− 50
α Hydræ	− 139	+ 28	− 3	− 118	− 80	+ 3
α Leonis	− 106	+ 43	+ 12	− 170	− 121	− 38
β Leonis	− 76	+ 34	+ 3	− 85	− 68	+ 19
α Virginis . . .	− 42	+ 28	− 3	− 86	− 54	+ 29
α Bootis	− 36	+ 39	+ 8	− 164	− 143	− 60
α^2 Libræ	+ 190	+ 23	− 8	+ 30	− 33	+ 50
α Coronæ . . .	− 66	+ 37	+ 6	− 240	− 208	− 125
α Serpentis . . .	+ 122	+ 35	+ 4	− 56	− 86	− 3
α Scorpii	+ 84	+ 4	− 27	+ 122	+ 98	+ 181
α Herculis . . .	− 16	+ 40	+ 9	− 225	− 202	− 119
α Ophiuchi . . .	− 162	+ 44	+ 13	− 161	− 116	− 33
α Lyræ	− 30	+ 28	− 3	− 127	− 108	− 25
γ Aquilæ. . . .	+ 163	+ 29	− 2	− 35	− 85	− 2
α Aquilæ. . . .	− 12	+ 18	− 13	− 50	− 37	+ 46
β Aquilæ. . . .	+ 141	+ 26	− 5	− 67	− 89	− 6
α^2 Capricorni. . .	− 17	+ 7	− 24	− 20	+ 1	+ 84
α Aquarii. . . .	− 115	+ 16	− 15	− 74	− 22	+ 61
α Pegasi	+ 7	+ 29	− 2	− 100	− 92	− 9
α Andromedæ . .	− 73	+ 45	+ 14	− 187	− 159	− 76

Corrections to right ascensions, &c.—Continued.

Star.	Maskelyne, 1805.			Auwers' Cacciotore.		
	c_3	c_3'	c_3''	c_4	c_4'	c_4''
γ Pegasi	− 148	− 152	− 24	+ 56	+ 52	+ 29
α Arietis	− 148	− 133	− 5	− 23	− 8	− 31
α Ceti	− 228	− 243	− 115	+ 27	+ 12	− 11
α Tauri	− 111	− 94	+ 34	+ 14	+ 31	+ 8
β Orionis	− 139	− 159	− 31	+ 61	+ 41	+ 18
β Tauri	− 62	− 101	+ 27	+ 110	+ 71	+ 48
α Orionis	− 123	− 129	− 1	+ 112	+ 106	+ 83
β Geminorum	− 189	− 161	− 33	+ 65	+ 93	+ 70
α Hydræ	− 133	− 106	+ 22	+ 28	+ 55	+ 32
α Leonis	− 173	− 136	− 8	+ 25	+ 62	+ 39
β Leonis	− 98	− 88	+ 40	+ 21	+ 31	+ 8
α Virginis	− 122	− 93	+ 35	− 29	0	− 23
α Bootis	− 97	− 82	+ 46	+ 22	+ 37	+ 14
α^2 Libræ	− 50	− 101	+ 27	− 2	− 53	− 76
α Coronæ	− 120	− 96	+ 32	− 74	− 50	− 73
α Serpentis	− 125	− 148	− 20	− 53	− 76	− 99
α Scorpii	− 206	− 223	− 95	+ 69	+ 52	+ 29
α Herculis	− 111	− 91	+ 37	− 81	− 61	− 84
α Ophiuchi	− 163	− 131	− 3	− 28	+ 4	− 19
α Lyræ	− 65	− 51	+ 77	− 5	+ 9	− 14
γ Aquilæ	− 71	− 111	+ 17	+ 54	+ 14	− 9
α Aquilæ	− 162	− 151	− 23	+ 44	+ 55	+ 32
β Aquilæ	− 120	− 135	− 7	+ 17	+ 2	− 21
α^2 Capricorni	− 148	− 126	+ 2	− 20	+ 2	− 21
α Aquarii	− 208	− 164	− 36	− 5	+ 39	+ 16
α Pegasi	− 140	− 133	− 5	+ 47	+ 54	+ 31
α Andromedæ	− 146	− 125	+ 3	+ 23	+ 44	+ 21

Corrections to right ascensions, &c.—Continued.

Star.	Bessel I, 1815.			Bessel II, 1825.		
	c_5	c_5'	c_5''	c_6	c_6'	c_6''
γ Pegasi	+ 140	+ 151	+ 98	0	+ 22	+ 27
α Arietis	+ 123	+ 120	+ 67	0	− 17	− 12
α Ceti	+ 126	+ 100	+ 47	0	− 35	− 30
α Tauri	+ 20	+ 27	− 26	0	0	+ 5
β Orionis	− 8	− 33	− 86	0	− 30	− 25
β Tauri	+ 32	+ 10	− 43	0	− 9	− 4
α Orionis	− 45	− 51	− 104	0	− 3	+ 2
β Geminorum	− 5	− 3	− 51	0	− 17	− 12
α Hydræ	− 78	− 82	− 135	0	− 28	− 23
α Leonis	− 16	− 2	− 55	0	− 4	+ 1
β Leonis	+ 34	+ 21	− 32	0	− 31	− 26
α Virginis	− 8	+ 14	− 39	0	+ 17	+ 22
α Bootis	+ 38	+ 40	− 13	0	− 8	− 3
α^2 Libræ	+ 166	+ 139	+ 86	0	− 8	− 3
α Coronæ	+ 102	+ 109	+ 56	0	− 6	− 1
α Serpentis	+ 83	+ 73	+ 20	0	+ 1	+ 6
α Scorpii	+ 121	+ 118	+ 65	0	+ 7	+ 12
α Herculis	+ 53	+ 66	+ 13	0	+ 7	+ 12
α Ophiuchi	+ 27	+ 20	− 33	0	− 37	− 32
α Lyræ	+ 56	+ 61	+ 8	0	− 2	+ 3
γ Aquilæ	+ 65	+ 44	− 9	0	− 6	− 1
α Aquilæ	+ 48	+ 55	+ 2	0	+ 4	+ 9
β Aquilæ	+ 63	+ 71	+ 18	0	+ 26	+ 31
α^2 Capricorni	+ 52	+ 76	+ 23	0	+ 26	+ 31
α Aquarii	+ 64	+ 37	− 16	0	+ 13	+ 18
α Pegasi	+ 112	+ 116	+ 63	0	+ 2	+ 7
α Andromedæ	+ 123	+ 123	+ 70	0	− 17	− 12

Corrections to right ascensions, &c.—Continued.

Star.	Struve.			Brinkley.		
	c_7	c_7'	c_7''	c_8	c_8'	c_8''
γ Pegasi	+ 11	+ 35	− 1	− 150	− 126	+ 9
α Arietis	+ 57	+ 38	+ 2	− 93	− 112	+ 23
α Ceti	+ 73	+ 37	+ 1	− 150	− 186	− 51
α Tauri	+ 22	+ 21	− 15	− 145	− 146	− 11
β Orionis	+ 119	+ 89	+ 53	− 82	− 112	+ 23
β Tauri	+ 20	+ 14	− 22	− 152	− 160	− 25
α Orionis	+ 28	+ 22	− 14	− 128	− 134	+ 1
β Geminorum	+ 34	+ 13	− 23	− 80	− 101	+ 34
α Hydræ	+ 93	+ 61	+ 25	− 6	− 38	+ 97
α Leonis	+ 39	+ 32	− 4	− 137	− 144	− 9
β Leonis	+ 58	+ 24	− 12	− 110	− 144	− 9
α Virginis	+ 4	+ 20	− 16	− 182	− 166	− 31
α Bootis	+ 29	+ 19	− 17	− 139	− 149	− 14
$α^2$ Libræ	+ 22	+ 17	− 19	− 126	− 131	+ 4
α Coronæ	+ 73	+ 66	+ 30	− 137	− 144	− 9
α Serpentis	+ 71	+ 74	+ 38	− 156	− 153	− 18
α Scorpii	+ 44	+ 52	+ 16	− 186	− 177	− 42
α Herculis	+ 67	+ 74	+ 38	− 115	− 108	+ 27
α Ophiuchi	+ 118	+ 76	+ 40	− 81	− 123	+ 12
α Lyræ	− 6	− 9	− 45	− 101	− 104	+ 31
γ Aquilæ	+ 50	+ 46	+ 10	− 146	− 150	− 15
α Aquilæ	+ 27	+ 30	− 6	− 148	− 145	− 10
β Aquilæ	+ 10	+ 39	+ 3	− 205	− 176	− 41
$α^2$ Capricorni	+ 37	+ 63	+ 27	− 189	− 163	− 28
α Aquarii	+ 21	+ 33	− 3	− 127	− 115	+ 20
α Pegasi	+ 5	+ 7	− 29	− 132	− 130	+ 5
α Andromedæ	− 6	− 25	− 61	− 87	− 106	+ 29

Corrections to right ascensions, &c.—Continued.

Star.	Argelander.			Pulkowa.		
	c_9	c_9'	c_9''	c_{10}	c'	c_{10}''
γ Pegasi	− 15	+ 14	. .	− 59	− 11	− 6
α Arietis	+ 28	+ 2	. .	+ 62	+ 8	+ 13
α Ceti.	+ 26	− 15	. .	+ 54	0	+ 5
α Tauri	+ 12	+ 8	. .	+ 6	− 16	− 11
β Orionis. . . .	+ 9	− 23	. .	+ 38	+ 2	+ 7
β Tauri	+ 13	+ 12	. .	− 17	+ 10	+ 15
α Orionis. . . .	+ 17	+ 11	. .	− 13	− 17	− 12
β Geminorum . .	+ 44	+ 14	. .	+ 70	+ 1	+ 6
α Hydræ	+ 44	− 1	. .	+ 90	0	+ 5
α Leonis	+ 12	0	. .	+ 50	− 3	+ 2
β Leonis	+ 42	− 1	. .	+ 75	0	+ 5
α Virginis . . .	− 26	− 13	. .	+ 3	+ 2	+ 7
α Bootis	+ 8	− 6	. .	+ 38	+ 5	+ 10
α² Libræ	− 8	− 4	. .	− 60	− 16	− 11
α Coronæ . . .	+ 36	+ 22	. .	+ 34	− 7	− 2
α Serpentis . . .	− 2	+ 6	. .	− 37	− 8	− 3
α Scorpii	− 55	− 41	. .	− 54	− 19	− 14
α Herculis . . .	+ 6	+ 10	. .	− 15	− 23	− 18
α Ophiuchi . . .	+ 73	+ 16	. .	+ 94	− 21	− 16
α Lyræ	+ 6	0	. .	+ 34	+ 12	+ 17
γ Aquilæ	+ 7	+ 10	. .	− 31	+ 5	+ 10
α Aquilæ	− 5	− 3	. .	+ 7	+ 1	+ 6
β Aquilæ	− 34	+ 4	. .	− 75	− 5	0
α² Capricorni . . .	− 58	− 31	. .	− 32	− 7	− 2
α Aquarii . . .	− 26	− 21	. .	+ 16	− 10	− 5
α Pegasi	+ 7	+ 8	. .	− 11	− 15	− 10
α Andromedæ . .	+ 41	+ 13	. .	+ 63	+ 1	+ 6

Corrections to right ascensions, &c.—Continued.

Star.	Greenwich, 1864.			Washington.		
	c_{11}	c_{11}'	c_{11}''	c_{12}	c_{12}'	c_{12}''
γ Pegasi	− 124	− 51	− 30	− 59	+ 22	− 12
α Arietis	+ 53	− 33	− 12	+ 123	+ 27	− 7
α Ceti.	+ 58	− 13	+ 8	+ 132	+ 56	+ 22
α Tauri	+ 40	+ 1	+ 22	+ 91	+ 48	+ 14
β Orionis. . . .	+ 24	− 18	+ 3	− 73	+ 29	− 5
β Tauri	− 58	0	+ 21	− 52	+ 16	− 18
α Orionis. . . .	+ 6	+ 3	+ 24	+ 38	+ 35	+ 1
β Geminorum . .	+ 111	− 3	+ 18	+ 151	+ 23	− 11
α Hydræ	+ 129	− 15	+ 6	+ 185	+ 23	− 11
α Leonis	+ 83	− 13	+ 8	+ 148	+ 39	+ 5
β Leonis	+ 110	− 4	+ 17	+ 160	+ 33	− 1
α Virginis . . .	− 22	− 38	− 17	+ 57	+ 37	+ 3
α Bootis	+ 39	− 17	+ 4	+ 97	+ 33	− 1
α^2 Libræ	− 93	− 5	+ 16	− 67	+ 34	0
α Coronæ . . .	+ 48	− 23	− 2	+ 98	+ 17	− 17
α Serpentis . . .	− 79	− 25	− 4	− 32	+ 29	− 5
α Scorpii	− 91	− 32	− 11	− 19	+ 48	+ 14
α Herculis . . .	+ 12	− 10	+ 11	+ 62	+ 35	+ 1
α Ophiuchi . . .	+ 178	− 5	+ 16	+ 246	+ 42	+ 8
α Lyræ	+ 9	− 29	− 8	+ 68	+ 25	− 9
γ Aquilæ	− 117	− 46	− 25	− 42	+ 40	+ 6
α Aquilæ	− 32	− 18	+ 3	+ 49	+ 34	0
β Aquilæ	− 148	− 39	− 18	− 89	+ 33	− 1
α^2 Capricorni. . .	− 54	− 29	− 8	0	+ 25	− 9
α Aquarii. . . .	+ 6	− 53	− 32	+ 115	+ 46	+ 12
α Pegasi	− 20	− 30	− 9	+ 60	+ 49	+ 15
α Andromedæ . .	+ 78	− 22	− 1	+ 151	+ 40	+ 6

The following are the values of x and y, in which the units are thousandths of seconds of time:

Star.	x	y
γ Pegasi	− 18	− 127
α Arietis	+ 12	+ 167
α Ceti	+ 32	+ 89
α Tauri	− 3	+ 93
β Orionis	+ 28	+ 32
β Tauri	+ 14	− 165
α Orionis	+ 5	− 5
β Geminorum . . .	+ 10	+ 236
α Hydræ	+ 19	+ 286
α Leonis	− 3	+ 225
β Leonis	+ 24	+ 206
α Virginis	− 19	+ 78
α Bootis	+ 4	+ 121
α^2 Libræ	+ 15	− 233
α Coronæ	+ 1	+ 160
α Serpentis	+ 3	− 129
α Scorpii	− 3	− 128
α Herculis	− 9	+ 72
α Ophiuchi	+ 26	+ 357
α Lyræ	− 1	+ 88
γ Aquilæ	+ 12	− 188
α Aquilæ	− 5	+ 39
β Aquilæ	− 19	− 206
α^2 Capricorni . . .	− 25	− 1
α Aquarii	− 18	+ 174
α Pegasi	− 3	+ 29
α Andromedæ . . .	+ 11	+ 199

§ 3.

DISCUSSION OF PROBABLE ERROR.

From these discordances we shall next deduce the probable error of the resulting x and y. It is well known that results for probable error generally prove entirely illusory. This does not arise from any defect in the theory of the subject, but from the fact that the probable error generally has to be deduced from the discordances among a series of results affected by some common source of error, which changes all the results in the same way, and is, therefore, not eliminated from the mean. But the twelve star-catalogues here employed may be regarded as absolutely independent of each other.

In deducing the probable error from each series of residuals, I take the mean value of the residuals without regard to the sign. In the more elegant method of deducing the probable error from the sum of the squares of the residuals, it seems to me that too much weight is assigned to those large residuals the presence or absence of which is a matter of chance. The probable error being proportional to the mean error, and only a little smaller, I shall use the latter throughout instead of the former. The mean values of the numbers found in the third columns, taken without regard to the sign, are as follows:

1. Auwers' Bradley	10.7
2. Piazzi	49
3. Maskelyne	30
4. Auwers' Cacciotore	36
5. Bessel I	47
6. Bessel II	13.7
7. Struve	21 1
8. Brinkley	23.3
9. Argelander	11.4
10. Pulkowa	8.2
11. Greenwich	13.1
12. Washington	7.9

Had the catalogues all been entirely accordant, these residuals would all have been zero. Let us next see in what manner they are made up. In the case of Bradley the residual is formed by subtracting from each c_1 the value of $x - 0.65\,y$ deduced from the expression on page 10, substituting for c_1, c_2, etc., their values found in the preceding table. Substituting in this expression these values of x and y in terms of c_1, c_2, etc., which have already been given, we find—

$$x - 0.65\,y = .68\,c_1 + .10\,c_2 + .09\,c_3 + .09\,c_4 + .06\,c_5 + .14\,c_6 + .13\,c_7 \\ + .04\,c_8 + .07\,c_9 - .07\,c_{10} - .12\,c_{11} - .22\,c_{12}$$

Subtracting this from c_1, the difference is—

$$.32\,c_1 - .10\,c_2 - .09\,c_3 - .09\,c_4 - .06\,c_5 - .14\,c_6 - .13\,c_7 - .04\,c_8 \\ - .07\,c_9 + .07\,c_{10} + .12\,c_{11} + .22\,c_{12} = c'$$

and it is this expression of which the value is given in the second column under each catalogue in the preceding tables.

Let us now represent by—

$$\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3\ \text{- - - - - -}\ \varepsilon_{12}$$

the probable errors of—

$$c_1,\ c_2,\ c_3\ \text{- - - - - -}\ c_{12}$$

or of the star-positions in each catalogue. The square of the mean error of a sum being equal to the sum of the squares of the individual mean errors, the square of the mean error of the last expression, or of c', will be—

$$.102\,\varepsilon_1^2 + .010\,\varepsilon_2^2 + .008\,\varepsilon_3^2 + .008\,\varepsilon_4^2 + .004\,\varepsilon_5^2 + .029\,\varepsilon_6^2 + .017\,\varepsilon_7^2 + .002\,\varepsilon_8^2 \\ + .005\,\varepsilon_9^2 + .005\,\varepsilon_{10}^2 + .014\,\varepsilon_{11}^2 + .048\,\varepsilon_{12}^2$$

The mean Bradley residual being 10.7, the value of this expression is 114.

A similar equation being formed for each of the twelve catalogues, we have twelve equations of the first degree, which suffice completely to determine the twelve values of

ε, and therefore the mean error of each catalogue. These equations are as follows, writing only the co-efficients of the unknown quantities:

ε_1^2	ε_2^2	ε	ε_4^2	ε_5^2	ε_6^2	ε_7^2	ε_8^2	ε_9^2	ε_{10}^2	ε_{11}^2	ε_{12}^2	
.102	.010	.008	.008	.004	.020	.017	.002	.005	.005	.014	.048	= 114
.109	.866	.004	.004	.002	.014	.014	.001	.006	.012	.0	.0	=2601
.084	.002	.902	.004	.002	.014	.012	.001	.000	.017	.000	.000	= 900
.084	.002	.004	.902	.002	.014	.012	.001	.0	.017	.0	.0	=1296
.036	.002	.002	.002	.922	.012	.012	.001	.006	.032	.002	.006	=2209
.022	.002	.002	.002	.002	.792	.012	.001	.006	.040	.005	.010	= 196
.020	.002	.002	.002	.002	.012	.792	.001	.006	.044	.005	.012	= 441
.020	.002	.002	.002	.002	.012	.012	.940	.006	.044	.005	.012	= 529
.012	.001	.001	.001	.002	.012	.012	.001	.846	.048	.006	.017	= 121
.001	.000	.000	.000	.001	.010	.010	.001	.006	.504	.017	.044	= 67
.029	.000	.000	.000	.001	.008	.010	.001	.006	.137	.672	.090	= 170
.048	.000	.000	.000	.000	.008	.010	.001	.006	.152	.036	.462	= 63

These equations will give the middle series of probable errors with great certainty; but those which lie near the extremes in point of time will be quite uncertain, the necessary uncertainty of the last members of each equation being considerably exaggerated in the result. If we solve by successive approximation, the equations for determining ε_1^2 and ε_{12}^2 will be—

$$.102\ \varepsilon_1^2 = 114 - 74 = 40; \quad \varepsilon_1 = 0^s.020$$
$$.462\ \varepsilon_{12}^2 = 63 - 55 = 8; \quad \varepsilon_{12} = 0^s.004$$

We may roughly estimate the uncertainty of the second members of the preceding equations as equal to one-fifth of their entire amount, so that the number for which we have found the value 114 might have ranged anywhere between 91 and 135, while that which we have found to be 63 might have ranged between 50 and 76. Hence the value of ε_1 may, from this cause alone, range between $0^s.013$ and $0^s.025$, and that of ε_{12} between zero and $0^s.007$. In the latter case we have an independent means of estimating the probable error. The catalogue result actually employed is the mean of two nearly independent determinations with different instruments. The mean difference of the results was $0^s.013$. We may therefore put $0^s.006$ as the inferior limit of ε_{12}. To make the estimate too great rather than too small, I have supposed ε_{12} to be $0^s.008$, and hence obtained the following series of probable errors, which nearly satisfy all the equations except the last:

s.

$\varepsilon_1 = .019$ $\quad\quad$ $\varepsilon_1^2 = 361$
$\varepsilon_2 = .054$ $\quad\quad$ $\varepsilon_2^2 = 2916$
$\varepsilon_3 = .030$ $\quad\quad$ $\varepsilon_3^2 = 900$
$\varepsilon_4 = .037$ $\quad\quad$ $\varepsilon_4^2 = 1369$
$\varepsilon_5 = .048$ $\quad\quad$ $\varepsilon_5^2 = 2304$
$\varepsilon_6 = .014$ $\quad\quad$ $\varepsilon_6^2 = 196$
$\varepsilon_7 = .023$ $\quad\quad$ $\varepsilon_7^2 = 529$
$\varepsilon_8 = .023$ $\quad\quad$ $\varepsilon_8^2 = 529$
$\varepsilon_9 = .010$ $\quad\quad$ $\varepsilon_9^2 = 100$
$\varepsilon_{10} = .010$ $\quad\quad$ $\varepsilon_{10}^2 = 100$
$\varepsilon_{11} = .014$ $\quad\quad$ $\varepsilon_{11}^2 = 196$
$\varepsilon_{12} = .008$ $\quad\quad$ $\varepsilon_{12}^2 = 64$

It may be remarked that the only uncertainty among these probable errors which materially affect our final result for the probable error of the concluded positions is that of ε_1.

To find the probable error of a computed star-position at the time t, we must substitute the values of x and y in the expression $x + yt$, which gives the correction to the *Tabulæ Regiomontanæ* at the time t. Calling this correction $\varDelta$ we have—

$$\begin{aligned}\varDelta = &+ (.17 - .78t)\, c_1\\ &+ (.04 - .10t)\, c_2\\ &+ (.04 - .08t)\, c_3\\ &+ (.04 - .08t)\, c_4\\ &+ (.04 - .03t)\, c_5\\ &+ (.11 - .04t)\, c_6\\ &+ (.11 - .03t)\, c_7\\ &+ (.03 - .01t)\, c_8\\ &+ (.08 + .01t)\, c_9\\ &+ (.19 + .40t)\, c_{10}\\ &+ (.06 + .27t)\, c_{11}\\ &+ (.09 + .47t)\, c_{12}\end{aligned}$$

The square of the probable error of this expression will be—

$$(.17 - .78t)^2\, \varepsilon_1^2 + (.04 - .10t)^2\, \varepsilon_2^2 + \text{etc.} = 36 - 125t + 303t^2$$

t being, it will be remembered, the fraction of a century after 1820. This expression would attain its minimum in 1840, when its value would be—

$$\varepsilon^2 = 23 \text{ or } \varepsilon = 0^{\text{s}}.0048$$

At any other time T its value will be—

$$\varepsilon^2 = 23 + 303 \left(\frac{\text{T} - 1840}{100}\right)^2$$

Owing to the independence of the twelve determinations on which the concluded positions rest, I conceive that this expression for the probable error is not entirely illusory, but is very nearly real. Of the exactness of the constant term no serious doubt can be entertained. But the co-efficient 303 multiplied by the square of the time depends very largely on the concluded probable error of Auwers' Bradley, and we have seen that there is some uncertainty in this determination. If this probable error is really as great as $0^{\text{s}}.025$, which we have seen to be quite possible, we should have—

$$\varepsilon^2 = 23 + 464 \left(\frac{\text{T} - 1841}{100}\right)^2$$

The first expression will give for the probable error in 1900—

$$\varepsilon = 0^{\text{s}}.011$$

while the second will give—

$$\varepsilon = 0^{\text{s}}.014$$

It must be understood that these results for probable error are true only on the assumption that the proper motion of each star carries it forward on a great circle with a uniform velocity.

§ 4.

SYSTEMATIC DISCORDANCES AMONG THE CATALOGUES.

Let us next inquire how far the outstanding residuals of the corrections to each catalogue are of a systematic character depending either upon the right ascension or declination. I have already set forth the reasons for believing that the systematic errors to be looked for are those depending on the sine and cosine of the right ascension. Let us then suppose the residuals of each catalogue to be of the form—

$$m \cos \alpha + n \sin \alpha + c$$

If we put r_1, r_2, etc., for the residuals corresponding to the several stars we shall have twenty-seven equations of the form—

$$m \cos \alpha_i + n \sin \alpha_i + c = r_i$$

from which to determine m, n, and c. If these equations be solved by least squares, the normal equations will be—

$$m \Sigma \cos^2 \alpha + n \Sigma \sin \alpha \cos \alpha + c \Sigma \cos \alpha = \Sigma r \cos \alpha$$
$$m \Sigma \sin \alpha \cos \alpha + n \Sigma \sin^2 \alpha + c \Sigma \sin \alpha = \Sigma r \sin \alpha$$
$$m \Sigma \cos \alpha + n \Sigma \sin \alpha + 27 c = \Sigma r$$

Owing to the minuteness of the quantities $\Sigma \cos \alpha$, $\Sigma \sin \alpha$, m, and n, and to the fact that when we take the residuals c_3 in the third column of the table of catalogues we have $\Sigma r = 0$, we may, using these same residuals, determine m and n from the first two equations, putting $c = 0$. Putting for $\Sigma \cos^2 \alpha$, etc., their values derived from the right ascensions of the twenty-seven stars in question at or near the mean epoch, these equations become—

$$11.74\, m + 0.93\, n = \Sigma r \cos \alpha$$
$$0.93\, m + 15.26\, n = \Sigma r \sin \alpha$$

which give—

$$11.68\, m = \Sigma r\, (\cos \alpha - .06 \sin \alpha)$$
$$15.19\, n = \Sigma r\, (\sin \alpha - .08 \cos \alpha)$$

Computing the values of m and n corresponding to each catalogue from these formulæ, we find that the residuals in the second column, which represent the excess of the catalogue right ascension over that computed, may be represented as follows:

No.	Catalogue.	Systematic residuals.
		$\quad s. \quad\quad s. \quad\quad\quad s.$
1	Auwers' Bradley . . .	$r = +.031 - .003 \cos \alpha + .007 \sin \alpha$
2	Piazzi, (1800)	$r = -.087 + .003 \cos \alpha - .001 \sin \alpha$
3	Maskelyne, (1805) . . .	$r = -.128 - .019 \cos \alpha - .006 \sin \alpha$
4	Auwers' Cacciotore . .	$r = +.023 + .010 \cos \alpha + .033 \sin \alpha$
5	Bessel I, (1815). . . .	$r = +.053 + .037 \cos \alpha - .037 \sin \alpha$
6	Bessel II, (1825) . . .	$r = -.005 + .006 \cos \alpha - .010 \sin \alpha$
7	Struve, (1825)	$r = +.036 - .007 \cos \alpha - .005 \sin \alpha$
8	Brinkley, (1825	$r = -.135 - .002 \cos \alpha + .009 \sin \alpha$
9	Argelander, (1828). . .	$r = .000 + .001 \cos \alpha + .002 \sin \alpha$
10	Pulkowa, (1845) . . .	$r = -.005 + .001 \cos \alpha + .002 \sin \alpha$
11	Greenwich, (1864) . . .	$r = -.021 - .011 \cos \alpha + .010 \sin \alpha$
12	Washington, (1870) . .	$r = +.034 + .004 \cos \alpha - .001 \sin \alpha$

The catalogues in which the periodic character of the residual is most strongly indicated are Auwers' Cacciotore, Bessel I, and Greenwich. In the case of the latter the systematic difference, small as it is, makes up the greater part of the residual, since, if it is subtracted, the mean value of the residual is reduced from .0131 to .0064, while the probable error is reduced in an even larger proportion. Taking away the three catalogues just mentioned, the co-efficients m and n are no greater than might result from the accumulation of accidental errors; it is therefore doubtful whether we ought to regard them as real.

To find what discrepancies depending on the declinations are indicated, I have grouped the twenty-seven stars as follows:

7 stars north of + 20°
11 stars between + 6° and + 16°
9 stars south of + 6°

The mean residual from the third column corresponding to each group is—

Catalogue.	North group.	Middle.	South.
	s.	*s.*	*s.*
Bradley	+ .006	.000	− .006
Piazzi	− .046	− .013	+ .049
Maskelyne . . .	+ .021	.000	− .022
Cacciotore. . . .	+ .005	+ .002	− .006
Bessel I	+ .013	− .006	− .004
Bessel II	− .006	+ .001	+ .004
Struve	− .019	+ .004	+ .010
Brinkley	+ .010	− .002	− .005
Argelander . . .	+ .008	+ .007	− .016
Pulkowa	+ .008	− .005	− .001
Greenwich. . . .	+ .003	+ .004	− .006
Washington . . .	− .008	+ .003	+ .003

Notwithstanding their general minuteness, these residuals depending on the declination are more certainly indicated than those depending on the right ascension. The catalogues in which they are largest, those of Piazzi and Maskelyne, have received but a small weight in the preceding discussion. In all the others, with the possible exception of Struve, they are so small that they may be treated as accidental errors. Their further consideration is therefore, for the present, omitted.

§ 5.

DETERMINATION OF THE POSITION AND MOTION OF THE EQUINOX.

For the determination of the common correction to the right ascensions of all stars on account of error in the assumed equinoctial point, we are still dependent entirely on observations of the sun. We may ultimately hope to determine some of the quantities on which the absolute positions of the stars depend from observations and theories of the planets, the orbit of each planet, when corrected for perturbations, being a fixed great circle, and the planets being susceptible of more accurate observation than the sun, but this course is not possible at present. It is well known that the best determinations of the equinox exhibit discordances far larger than can result from purely accidental errors of observation, and so indicate the existence of systematic errors in the observations of the sun, the nature and cause of which are obscure. The determination of each co-ordinate of the sun seems about equally liable to them. We know that the absolute right ascension of a star depends effectively upon two elements, to be determined from observation, namely:

1. The moment at which the sun crossed the equinox, from which its absolute right ascension at any time may be obtained, the elements of its orbit being known.

2. The difference of right ascension between the sun and a star, from which the absolute right ascension of the latter is obtained from that of the former.

Since any error depending on the sun's zenith distance will, in the first determination, be eliminated from the mean of the two equinoxes, it might be supposed that there would be a general agreement among different authorities in this determination. But such is by no means the case; and I can assign no probable cause for the observed discrepancies.

With reference to the second determination, it is well known that the personal equation of an observer is frequently different for the sun and for a star, and this difference will appear in the difference of their right ascensions. Diurnal changes in the clock, the instrument, and the personal equation of the observer will also introduce errors into the determination.

Under these circumstances the most we can do is to base our determination on the widest range of material at our disposal, depending for accuracy on the number and variety of the determinations employed rather than upon the precision of any one. All the data at my disposal which did not involve a re-reduction of observations are comprised in the following discussion.

The different common corrections to the preceding determination of right ascensions of stars are represented by *e*. We begin with an examination of those published catalogues in which the star-positions are referred directly to the sun.

Bessel's Bradley.

The right ascensions of the following stars were determined by Bessel by direct comparison with the sun. Opposite each star is given Auwers' correction to Bessel's position, and the concluded correction already found, the signs being changed:

Star.	B. – A.	B – (1).
	.	s.
α Tauri	+ .012	+ .063
β Orionis	– .040	– .007
α Orionis	– .030	– .008
β Geminorum	+ .102	+ .143
α Leonis	+ .106	+ .149
α Virginis	+ .042	+ .070
α Bootis	+ .036	+ .075
α Lyræ	+ .030	+ .058
α Aquilæ	+ .012	+ .030
Mean	+ .030	+ .064

We have, therefore, from Bessel's determination for 1755—

$$e = +0^s.064$$

PIAZZI, 1800.

Of Piazzi's determination of the equinox I know nothing. The mean correction given by his catalogue cannot, therefore, receive much weight, although the geographical position of his observatory is very favorable to its determination. The value of e, resulting from this and from all the other original catalogues employed in the preceding determination is the same with the constant terms in the systematic residuals, given on page 21, and represented by r.

MASKELYNE, 1805.

Maskelyne states that his catalogue was "settled with respect to the equinoctial points, from the observed declinations of the sun about the equinoxes in 1804, 1805, and 1806." With the same catalogue he gives one for 1756, deduced from Bradley's observations, the mean of which is $0^s.10$ less than Bessel's, if we compare the twenty-seven stars under consideration, or $0^s.15$ less, if we compare only the stars which Bessel found by direct comparison with the sun. This large difference may arise from Maskelyne's having used only observations of the sun very near the equinoxes, while Bessel extended his determination to a declination of 14° on each side. An examination of the tables of corrections corresponding to different declinations of the sun, given on page 15 of the Fundamenta, does, in fact, show a systematic difference depending on the declination, but their source and nature are too obscure to admit of much discussion. All we can do is to suppose Maskelyne's determination an independent one, entitled only to small weight.

BESSEL, 1815 AND 1825.

Both Bessel's catalogues are referred directly to the sun. The first depends on three years' observations with the old transit and the Cary circle, numbering 290 in all; the second on five years' observations with the Reichenbach meridian-circle, numbering about 400 in all.

STRUVE, 1825.

Struve's equinox depends on 216 observations of the sun, extending through a period of four years. (Positiones Mediæ, p. xxxx.)

ARGELANDER, 1830.

From a remark in the introduction to his catalogue, page iii, it would appear that Argelander, from his own observations of the sun, found a correction of $+0^s.072$ to the right ascensions of the Tabulæ Regiomontanæ. We have, therefore, on this authority—

$$e = +0^s.064; \text{ epoch, } 1828.$$

POND, 1830.

In the introduction to his "Catalogue of 1112 stars," Pond states that the right ascensions of Maskelyne's last catalogue have been increased by $0^s.20$, being a quantity determined by observations of the sun made about the equinoxes with the present transit-instrument since 1816. The mean value of e given by this catalogue is $+0^{s}.073$. But, as the mean epoch of the observations is about seven years earlier, we may inquire whether the correction is different for this epoch. A correction of $+0^{s}.20$ applied to Maskelyne's catalogue for 1805 would leave—

$$e = +.072$$

I infer, therefore, that the correction should have the same value for the mean epoch, which we may consider to be 1825.

AIRY'S GREENWICH CATALOGUES.

The five catalogues published by Airy have been so carefully referred to the sun that they do not appear to need any further discussion. The comparison of the individual catalogues with the positions already found give the following values of e:

		s.
First six-year catalogue	epoch 1839,	$e = -.052$
Second six-year catalogue	1845,	$e = -.003$
Third six-year catalogue	1851,	$e = +.017$
First seven-year catalogue	1857,	$e = -.005$
Second seven-year catalogue	1864,	$e = -.021$

PULKOWA.

The Pulkowa equinox depends on observations of the sun with the transit-instrument and the vertical circle, extending from the year 1842 to 1850. The copious discussion of these observations by Wagner is found in "Observations de Poulkova," vol. III, pp. 41–67.

Adding Washington, 1870, this completes the list of original catalogues referred independently to the sun. But we have still a collection of data in the form of observations of the sun made and reduced at various observatories, and compared with the tables. I have reduced all these in a summary manner, which, though coarse and rude, will, I am persuaded, lead to results differing from those of the most refined discussion only by quantities much smaller than the unavoidable probable errors of the results. The mean correction to the tabular right ascension and declination was taken for each month through a series of years, all the observations made in any month of a given name—January, for instance—being combined into a single mean. This mean was that of the entire number of individual results. The correction to the sun's absolute mean right ascension was then deduced from the observed errors of the tabular declination, being given by the approximate formula—

$$\delta \text{ R. A. (in time)} = \tfrac{1}{36} \Sigma \Delta \delta \cos l$$

l being the sun's longitude, and the factor $\cos l$ being the mean value of the cosine of the sun's true longitude during each month.

The mean correction of the sun's absolute right ascension thus obtained is compared with the correction obtained by comparison with the stars, and the difference, supposed to be due to error in the adopted right ascensions of the latter, gives the mean correction to the equinox to obtain the absolute right ascension of the stars.

PARIS, 1854–'63.—(*Corrections to Carlini's tables.*)

Month.	$\Delta\alpha$	No. of obs.	δ	No. of obs.	cos l	δ cos l
	s.		″			″
January . .	+ .132	56	+ 1.07	43	+ .44	+ 0.47
February . .	+ .154	86	+ 2.76	73	+ .81	+ 2.24
March . . .	+ .165	104	+ 2.68	98	+ .98	+ 2.63
April . . .	+ .209	116	+ 2.25	105	+ .89	+ 2.00
May . . .	+ .245	80	+ 1.78	77	+ .56	+ 1.00
June . . .	+ .234	81	+ 1.20	74	+ .10	+ 0.12
July . . .	+ .207	120	+ 0.80	84	— .39	— 0.31
August . .	+ .213	126	+ 0.19	95	— .79	— 0.15
September .	+ .194	98	— 0.31	76	— .98	+ 0.30
October . .	+ .133	93	+ 0.50	68	— .92	— 0.46
November .	+ .196	72	+ 0.11	52	— .60	— 0.07
December .	+ .148	49	+ 1.30	40	— .10	— 0.13
Sum . .	+ 2.230	.	. .	.	. .	+ 7.64

We hence deduce—

	s.
Mean correction to sun's absolute right ascension	+0.212
Mean correction deduced from stars	+0.186
Therefore star's right ascensions require the correction . .	+0.026

A comparison of the corrections to the Tabulæ Regiomontanæ adopted in the reduction of the Paris observations with those deduced in this paper show that the latter are greater by $0^s.024$. We may therefore put as the result of this series of Paris observations—

$$e = 0^s.000$$

PARIS, 1864–'65—(*Corrections to Le Verrier's tables.*)

Month.	$\Delta\alpha$	No. of obs.	δ	No. of obs.	$\Delta\delta$ cos l
	s.		″		″
January . .	— .072	10	+ 0.20	7	+ 0.09
February . .	— .060	7	+ 1.35	6	+ 1.11
March . . .	+ .028	16	+ 1.08	16	+ 1.06
April . . .	— .030	24	+ 1.39	22	+ 1.24
May . . .	— .013	21	— 0.05	20	— 0.03
June . . .	— .052	19	— 0.29	18	— 0.03
July . . .	— .064	14	— 0.88	16	+ 0.34
August . .	— .022	13	+ 0.11	14	— 0.09
September .	— .006	23	+ 0.52	25	— 0.51
October . .	— .022	19	+ 0.45	16	— 0.41
November .	+ .037	15	— 1.48	14	+ 0.89
December .	— .017	13	— 0.86	13	+ 0.09
Sum . .	— 0.293	.	. .	.	+ 3.75

We hence deduce—

	s.
Mean correction to sun's absolute right ascension	+ 0.104
Mean correction deduced from stars	— 0.024
Correction to adopted right ascensions of stars	+ 0.128

The Paris concluded right ascensions for 1864 are less than those here deduced by $0^{s}.013$. The result of these two years' work at Paris is therefore—

$$e = +0^{s}.115$$

In the following two years nearly all the observations of the sun at Paris were made by a single observer, whose results are remarkably different from those of other observers, as may be seen by the very large and uniformly positive corrections to the right ascension and longitude of the sun, given in the collected results of the observations. This error evidently arises from the personal equation of the observer not being the same for the sun and for a star. Such differences are known to exist, but they are generally so minute as hardly to admit of positive detection. We have, however, one or two cases of the differences attaining an extraordinary magnitude, and, as they do not follow the usual law of error in their distribution among different observers, we must reject entirely the cases in question.

CAMBRIDGE, (ENGLAND.)

The results of the observations of the sun during the five years 1833–'37 are as follows:

Month.	Δ α	No. of obs.	Δ δ	No. of obs.	Δ δ cos l
	s.		"		"
January	+ .203	31	+ 1.00	30	+ 0.44
February	+ .195	40	+ 0.96	41	+ 0.80
March	+ .175	44	+ 0.18	44	+ 0.18
April	+ .230	55	+ 0.36	56	+ 0.32
May	+ .215	96	+ 0.68	95	+ 0.38
June	+ .138	71	+ 0.35	77	+ 0.04
July	+ .205	59	+ 0.66	58	— 0.26
August	+ .182	77	+ 0.14	80	— 0.11
September	+ .208	65	+ 0.07	65	— 0.07
October	+ .180	73	+ 0.46	75	— 0.43
November	+ .237	61	+ 0.79	59	— 0.47
December	+ .175	42	+ 0.93	37	— 0.09
Sum	+ 2.343	.	. .	.	+ 0.73

	s.
Mean correction to sun's absolute right ascension	+ 0.020
Correction deduced from stars	+ 0.195
Correction to adopted right ascension of stars	— 0.175
Adopted right ascensions *greater* than ours by	0.146

Hence—

$$e = -0^{s}.03;\ \text{epoch, } 1835.$$

In subsequent years the discussion of the correction for each year in which sufficient observations of the sun were made is given along with the results of the observations. The following are the mean values of e for series, each including five years' actual observation:

$$\begin{aligned} & \qquad\quad s. \\ 1841,\ e &= +0.06 \\ 1849,\ e &= +0.06 \\ 1856,\ e &= -0.01 \end{aligned}$$

OXFORD.

Since 1861 the sun has been pretty regularly observed at the Radcliffe Observatory, and the apparent errors of the tabular positions are given in the results of each year's observations. They are divided into two series, the first comprising the years 1861–'63, in which the results are compared with Carlini's tables, and the observations are considerably scattered; the second, comprising the years 1864–'68, during which the comparison has been made with Le Verrier's tables.

The tabular summary of the results for the years 1861–'63 is as follows:

Month.	Δ α	No. of obs.	Δ δ	No. of obs.	Δ δ cos l
	s.		″		″
January	+ .18	9	+ 0.2	12	+ 0.1
February	+ .40	14	+ 0.6	14	+ 0.5
March	+ .34	12	+ 0.6	15	+ 0.6
April	+ .32	19	+ 1.4	19	+ 1.3
May	+ .33	16	+ 1.5	11	+ 0.8
June	+ .16	13	+ 0.6	11	+ 0.1
July	+ .30	16	+ 0.5	17	− 0.2
August	+ .38	8	+ 1.4	8	− 1.1
September	+ .35	14	− 1.3	14	+ 1.3
October	+ .53	7	+ 0.8	9	− 0.7
November	+ .46	15	+ 0.7	15	− 0.4
December	+ .40	12	+ 2.3	12	− 0.2
Sum	+ 4.15	.	. .	.	+ 2.1

	s.
Mean correction to sun's absolute right ascension	+0.06
The same deduced from stars	+0.35
Correction to adopted right ascension of stars	−0.29

These adopted right ascensions are the same as those used during the corresponding year in the reduction of the Greenwich observations, the mean of which corresponds very nearly to our adopted results. We have, therefore, from these Radcliffe observations—

$$e = -0^{s}.29;\ \text{epoch, } 1862.$$

The great and improbable magnitude of this result renders the question of its introduction somewhat embarrassing.

For the five subsequent years, 1864–'68, we have—

Month.	$\Delta \alpha$	No. of obs.	$\Delta \delta$	No. of obs.	$\Delta \delta \cos l$
	s.		"		"
January . .	+ .11	36	+ 2.0	36	+ 0.9
February . .	+ .10	37	+ 1.0	37	+ 0.8
March . . .	+ .13	28	+ 1.0	27	+ 1.0
April . . .	+ .17	43	− 0.1	45	− 0.1
May . . .	+ .15	45	− 0.1	47	− 0.1
June . . .	+ .08	56	− 0.7	55	− 0.1
July . . .	+ .06	55	+ 0.3	57	− 0.1
August . .	+ .08	47	+ 0.7	47	0.6
September . .	+ .08	52	+ 0.1	50	− 0.1
October . .	+ .10	46	− 0.2	47	+ 0.2
November . .	+ .12	42	+ 1.0	41	− 0.6
December . .	+ .10	28	+ 1.4	28	− 0.1
Sum . .	+ 1.28	.	. .	.	+ 1.1

	s.
Correction to sun's absolute right ascension . . .	+ 0.03
Correction deduced from stars	+ 0.11
Correction to adopted right ascension of stars . .	− 0.08

The adopted right ascensions being still the same as at Greenwich, this result gives—

$$e = -0^{s}.09; \text{ epoch, } 1866.$$

The correction is still negative, as in the years 1861–'63, though not so great.

A comparison of the Radcliffe results with those derived from the Greenwich observations shows that the difference is found chiefly in the observed relative right ascensions of the sun and stars. Thus, during the years 1861–'63, the mean correction to the sun's right ascension deduced from the Greenwich observations is $+0^{s}.17$, while that from the Radcliffe observations, using the same adopted right ascensions of stars, is $+0^{s}.35$, so that the Radcliffe results are greater by $0^{s}.18$.

An examination of the Radcliffe observations shows that this great discordance did not exist in the year 1861, but commenced in 1862, with the introduction of the new transit-circle. It might, therefore, well give rise to the suspicion that the pointing of the circle is affected with a diurnal change. As the clock-error is determined from observations made at night, a constant error would thus be introduced into the results of transits of the sun. But this explanation seems to be negatived by the fact that the observations of the moon, though made at the same time with the stars, deviate still more widely from the truth. In fact, the mean correction to Hansen's right ascension, given by the observations of 1862, is $+0^{s}.62$, while all other observations show that the right ascension was then very nearly correct. We may hence conclude that the cause is to be sought for in some personal peculiarities of the observers. Though this explanation is made unsatisfactory by the circumstance that the observations of the two principal observers lead to nearly the same result, it is noticeable that in the year 1863 we find four observations of the moon signed M., probably made by the director of the observatory, the mean correction from which is $+ 0^s.035$. This seems, therefore, to be the only admissible explanation.

EDINBURGH.

In the Edinburgh observations for the years 1836–'39 the correction to the equinox is deduced from each year's observations. The mean result is—

$$e = + 0^s.04; \text{ epoch, } 1838.$$

Observations of the sun were continued till 1843. The published corrections to the tables lead to the following mean results:

Month.	$\Delta\alpha$	No. of obs.	$\Delta\delta$	No. of obs.	$\Delta\delta \cos l$
	s.		″		″
January . .	+ .17	50	+ 1.3	32	+ 0.6
February . .	+ .12	37	+ 1.3	41	+ 1.1
March . . .	+ .09	53	+ 0.4	60	+ 0.4
April . . .	+ .08	57	+ 0.5	50	+ 0.4
May. . . .	+ .13	42	− 0.1	46	− 0.1
June . . .	+ .11	51	+ 0.8	47	+ 0.1
July. . . .	+ .09	37	− 0.2	43	+ 0.1
August. . .	+ .05	43	− 0.2	44	+ 0.2
September. .	+ .08	44	− 1.0	43	+ 1.0
October . .	+ .18	32	− 0.3	46	+ 0.3
November. .	+ .05	32	+ 0.3	20	− 0.2
December. .	+ .13	38	− 0.3	41	0.0
Sum . .	+ 1.28	.	. .	.	+ 3.9

	s.
Mean correction to sun's absolute right ascension . .	+ 0.108
Mean correction deduced from stars	+ 0.106
Correction to adopted right ascensions	+ 0.002

The adopted right ascensions are $0^s.01$ less than those in this paper; therefore—

$$e = - 0^s.01; \text{ epoch, } 1842.$$

BERLIN.

The introduction to the first volume of the Berlin observations, published by Encke, gives the results of observations of the sun from 1839 to 1842. The following are the mean results:

Month.	$\Delta\alpha$	No. of obs.	$\Delta\delta$	No. of obs.	$\Delta\delta\cos l$
	s.		″		″
January	— .14	4	— 1.4	5	— 0.6
February	+ .04	20	0.0	20	0.0
March	+ .12	5	+ 0.2	4	+ 0.2
April	+ .06	18	+ 0.9	18	+ 0.8
May	+ .02	14	— 0.4	14	— 0.2
June	.00	7	— 1.1	7	— 0.1
July	— .03	7	+ 2.4	6	— 0.9
August	+ .12	11	+ 0.1	11	— 0.1
September	+ .07	25	0.0	23	0.0
October	+ .04	13	— 1.3	10	+ 1.2
November	+ .11	14	+ 0.7	14	— 0.4
December	+ .11	7	+ 0.2	8	0.0
Sum	+ .52				— 0.1

	s.
Mean correction to absolute right ascension	— 0.003
Correction from stars	+ 0.043
Correction to right ascension of stars	— 0.046

The adopted right ascensions are those of the Tabulæ Regiomontanæ, which at this time are $0^{s}.014$ less than those deduced. We have, therefore,

$$e = -0^{s}.06;\ \text{epoch, } 1841.$$

The Berlin observations are distributed among the months with unusual irregularity. Had the mean by weights been taken, the result would have been—

$$e = -0^{s}.07$$

WASHINGTON.

The following are the mean results of observations made with the transit-instrument and mural circle during the four years 1862–'65:

Month.	$\Delta\alpha$	No. of obs.	$\Delta\delta$	No. of obs.	$\Delta\delta\cos l$
	s.		″		″
January	— 0.052	42	+ 0.17	34	+ 0.07
February	— 0.045	42	— 0.30	30	— 0.24
March	— 0.022	41	+ 0.40	31	+ 0.39
April	+ 0.022	41	+ 0.70	36	+ 0.62
May	— 0.020	53	+ 0.22	53	+ 0.12
June	+ 0.008	50	+ 0.25	66	+ 0.02
July	+ 0.015	51	+ 0.30	55	— 0.12
August	— 0.002	57	0.00	50	0.00
September	— 0.038	42	— 0.22	47	+ 0.22
October	— 0.033	35	— 0.10	55	+ 0.09
November	— 0.027	35	— 0.10	46	+ 0.06
December	— 0.052	27	+ 0.57	32	— 0.06
Sum	— 0.238				+ 1.17

	s.
Mean correction to sun's absolute right ascension - -	+0.032
Correction deduced from stars - - - - - - - - -	−0.020
Correction to adopted right ascension of stars - - -	+0.052

The adopted right ascensions are those of Dr. Gould, the mean of which, at this epoch, scarcely differs from that of our deduced positions. We have, therefore, from these observations—

$$e = +0^s.05; \text{ epoch, } 1864.$$

Since 1866 the observations have been made with the transit-circle, but the instrument was dismounted a part of the time. The first two years' work gave the discordant results—

		s.
1866 - - - -	$e =$	$+0.07$
1867 - - - -	$e =$	0.00
Mean - - - -	$e =$	$+0.035$

The following are the mean results for 1868–'69:

Month.	Δα	No. of obs.	Δδ	No. of obs.	Δδ cos l
	s.		″		″
January . .	+ 0.10	10	+ 0.8	8	+ 0.4
February . .	+ 0.04	20	+ 0.3	19	+ 0.2
March. . .	+ 0.05	20	+ 0.7	18	+ 0.7
April . . .	+ 0.06	15	+ 0.7	19	+ 0.6
May . . .	+ 0.02	22	0.0	24	0.0
June . . .	+ 0.06	13	− 0.2	13	0.0
July . . .	+ 0.07	15	− 0.3	15	+ 0.1
August . .	+ 0.06	9	+ 0.5	7	− 0.4
September .	+ 0.03	9	− 0.7	9	+ 0.7
October . .	− 0.02	13	− 1.5	14	+ 1.4
November .	+ 0.01	12	− 0.7	12	+ 0.4
December .	+ 0.06	10	− 0.9	10	+ 0.1
Sum . .	+ 0.54	. .	. . . .	. .	+ 4.2

	s.
Mean correction to sun's absolute right ascension - -	+0.117
Correction deduced from stars - - - - - - - - -	+0.045

Whence—

$$e = +0^s.07$$

The comparison is with Hansen's tables, the mean right ascension of which is about 0ˢ.06 or 0ˢ.07 less than that of Le Verrier's tables.

The true value of e being of the form $x' + y'T$, every separate determination of its value gives an equation of condition between x' and y'. The following is the system of equations thus obtained. I have included the mean result of the Greenwich observations for 1868, 1869, and 1870, which have been published since the appearance of the last Greenwich seven-year catalogue. The Radcliffe results are omitted.

The weights have generally been assigned with reference to the entire number of observations on which each result depends, and also on the number of years over which the observations extend. But they have not been supposed to increase proportionally to either the time or the number of observations, so that, although the weight 1 would be assigned to the result of a single average year's work with the best instruments, the weight assigned to seven consecutive years' work by the same observers, and with the same instruments, is much less than 7. On the same principle, greater proportional weight has been given to isolated determinations than to determinations founded on long series of observations.

Observations.	Year.	Equation.	Years.	Weight.
		s.		
Bessel's Bradley	1756	$x' - .64y' = +.06$	7	3
Piazzi	1800	$x' - .20y' = -.08$	(?	2
Maskelyne	1805	$x' - .15y' = -.13$	2	1
Bessel I	1817	$x' - .03y' = +.05$	3	2
Bessel II	1822	$x' + .02y' = \quad .00$	5	4
Pond	1823	$x' + .03y' = +.07$	14	4
Struve	1825	$x' + .05y' = +.04$	5	4
Argelander	1828	$x' + .08y' = +.06$	4	3
Cambridge	1835	$x' + .15y' = -.03$	5	2
Edinburgh	1838	$x' + .18y' = +.04$	4	2
Airy	1839	$x' + .19y' = -.05$	6	3
Berlin	1841	$x' + .21y' = -.07$	4	2
Cambridge	1841	$x' + .21y' = +.06$	5	2
Edinburgh	1842	$x' + .22y' = -.01$	4	2
Pulkowa	1845	$x' + .25y' = \quad .00$	8	5
Greenwich	1845	$x' + .25y' = \quad .00$	6	3
Cambridge	1849	$x' + .29y' = +.06$	5	2
Greenwich	1851	$x' + .31y' = +.02$	6	3
Cambridge	1856	$x' + .36y' = -.01$	5	2
Greenwich	1857	$x' + .37y' = -.01$	7	3
Paris	1859	$x' + .39y' = \quad .00$	10	4
Greenwich	1864	$x' + .44y' = -.02$	7	3
Washington	1864	$x' + .44y' = +.05$	4	4
Paris	1865	$x' + .45y' = +.11$	2	2
Washington	1867	$x' + .47y' = +.05$	4	4
Greenwich	1869	$x' + .49y' = -.03$	3	3

Treating these equations by the method of least squares, we are led to the normal equations—

$$\begin{aligned} 74\ x' + 14.9\ y' &= +1^{s}.02 \\ 14.9\ x' + 7.4\ y' &= +0^{s}.17 \end{aligned}$$

the solution of which gives—

$$\begin{aligned} x' &= +0^{s}.016 \\ y' &= -0^{s}.009 \end{aligned}$$

The common correction to be applied to the star-positions first deduced is therefore—

$$+0^s.016 - .009\,T$$

T being the fraction of a century after 1820. Applying these constant corrections to the values of x and y previously found, we shall have a set of corrections to the Tabulæ Regiomontanæ which we may suppose to be free from all systematic error whatever, at least so far as it is possible to free them with our existing data.

§ 6.

CORRECTIONS FOR THE TERMS MULTIPLIED BY THE SECOND AND THIRD POWERS OF THE TIME.

We have hitherto proceeded on the supposition that the necessary corrections to the Tabulæ Regiomontanæ increase uniformly with the time. But, on computing the co-efficients of the second and third powers of the time, in the expression for the mean right ascension of the star at any epoch, and comparing them with those found by induction from the mean places of the Tabulæ Regiomontanæ, it is found that this supposition is not correct. In fact, it appears that Bessel computed the second differential or secular variation of the annual variation without taking any account of the effect of proper motion, and that he considered it constant, thus neglecting the third derivative entirely.

The following table shows, 1, the secular variation of the precession actually employed in the Tabulæ Regiomontanæ; 2, the adopted secular variation of the *annual motion* for 1800, computed from the co-efficients of Struve and Peters, using Hill's formulæ in the "Star Tables of the American ephemeris," page 19; 3, the resulting correction to the first value; 4, the variation of the secular variation in one century, or the value of $\frac{d^3\,\text{R. A.}}{d\,\text{T}^3}$, the unit of right ascension being $0^s.001$, and the unit of T one century; 5 and 6, the corrections to x and y necessitated by the addition of these terms to the positions of the Tabulæ Regiomontanæ.

These last terms are derived as follows: Representing, for brevity, by a and b the corrections to Bessel's assumed values of $\frac{d^2\,\text{R. A.}}{d\,\text{T}^2}$ and $\frac{d^3\,\text{R. A.}}{d\,\text{T}^3}$, the right ascensions of the Tabulæ Regiomontanæ should have received the correction—

$$\tfrac{1}{2}\,a\,\text{T}^2 + \tfrac{1}{6}\,b\,\text{T}^3$$

for the epoch of each catalogue before the comparison with the catalogue. T being here counted from 1800, the values of c_1, c_2, etc., already given, would have been thus changed by the following amounts:

$c_1 = -0.10a + 0.02b$

c_2 to $c_4 = 0.$

$c_5 = -.01a$

$c_6 = -.03a$

$c_7 = -.03a$

$c_8 = -.03a$

$c_9 = -.04a$

$c_{10} = -.10a - .02b$

$c_{11} = -.20a - .04b$

$c_{12} = -.24a - .06b$

Substituting these values of c_1, etc., in the expressions for x and y, (p. 10,) we find—

$$\Delta x = -.08\,a - .01\,b$$

$$\Delta y = -.13\,a - .06\,b$$

the values of which are given in the last two columns.

Star.	Bessel's $\frac{d^2 \text{ R. A.}}{d\,T^2}$	From Struve, (1800.)	a	b	Δx	Δy
γ Pegasi	+ 970	+ 958	− 12	+ 63	0	− 2
α Arietis	+ 2010	+ 1998	− 12	+ 41	0	− 1
α Ceti	+ 970	+ 959	− 11	+ 14	+ 1	+ 1
α Tauri	+ 1090	+ 1065	− 25	− 39	+ 2	+ 6
β Orionis	+ 430	+ 407	− 23	− 14	+ 2	+ 4
β Tauri	+ 890	+ 862	− 28	− 95	+ 3	+ 9
α Orionis	+ 310	+ 303	− 7	− 42	+ 1	+ 4
α Canis Majoris	+ 40	− 64	− 104	− 11		
α Canis Minoris	− 440	− 515	− 75	− 20		
β Geminorum	− 1220	− 1216	+ 4	− 64	0	+ 4
α Hydræ	− 150	− 157	− 7	+ 25	0	− 1
α Leonis	− 1020	− 1039	− 19	+ 37	+ 1	0
β Leonis	− 770	− 768	+ 2	+ 63	− 1	− 4
α Virginis	+ 1120	+ 1111	− 9	+ 50	0	− 2
α Bootis	+ 120	+ 202	+ 82	+ 44	− 7	− 13
α^2 Libræ	+ 1550	+ 1537	− 13	+ 17	+ 1	+ 1
α Coronæ	+ 240	+ 228	− 12	+ 17	+ 1	+ 1
α Serpentis	+ 630	+ 609	− 21	+ 5	+ 2	+ 2
α Scorpii	+ 1570	+ 1549	− 21	− 56	+ 2	+ 6
α Herculis	+ 370	+ 353	− 17	− 11	+ 1	+ 3
α Ophiuchi	+ 350	+ 344	− 6	− 13	+ 1	+ 2
α Lyræ	+ 160	+ 108	− 52	− 6	+ 4	+ 7
γ Aquilæ	− 80	− 99	− 19	− 5	+ 2	+ 3
α Aquilæ	− 140	− 180	− 40	− 7	+ 3	+ 5
β Aquilæ	− 150	− 140	+ 10	− 8	− 1	− 1
α^2 Capricorni	− 810	− 834	− 24	− 18	+ 2	+ 4
α Aquarii	− 430	− 444	− 14	+ 35	+ 1	0
α Pegasi	+ 530	+ 523	− 7	+ 56	0	− 2
α Andromedæ	+ 1770	+ 1770	0	+ 83	− 1	− 5

The values of x and y, or the correction to the right ascension and annual variation of the Tabulæ Regiomontanæ for the epoch 1820, may now be found by adding to the values of those quantities already found (p. 17) the constants for correction of equinox + 16 and − 9, and the corrections from the above table. After doing this we have, for convenience of application, transferred the epoch to 1800. Adding the co-efficients for the second and third powers of the time, we have the following values of the corrections in thousandths of seconds of time to the right ascensions of the Tabulæ Regiomontanæ, where we put T = fraction of century after 1800:

Corrections to Tabulæ Regiomontanæ.

γ Pegasi,	⊿ R. A. =	+ 26	$-138T$	$-6T^2$	$+10T^3$	
α Arietis,		− 3	+ 157	− 6	+ 7	
α Ceti,		+ 33	+ 81	− 6	+ 2	
α Tauri,		− 3	+ 90	− 12	− 6	
β Orionis,		+ 41	+ 27	− 12	− 2	
β Tauri,		+ 66	− 165	− 14	− 16	
α Orionis		+ 24	− 10	− 4	− 7	
α Canis Majoris,		+ 26	+ 64	− 52	− 2	+ *q* (Auwers)
α Canis Minoris,		− 27	+ 127	− 38	− 3	+ *q* (Auwers)
β Geminorum,		− 20	+ 231	+ 2	− 11	
α Hydræ,		− 20	+ 276	− 4	+ 4	
α Leonis,		− 29	+ 216	− 10	+ 6	
β Leonis,		0	+ 193	+ 1	+ 10	
α Virginis,		− 16	+ 67	− 4	+ 8	
α Bootis		− 7	+ 99	+ 41	+ 7	
α^2 Libræ,		+ 80	− 241	− 6	+ 3	
α Coronæ,		− 12	+ 152	− 6	+ 3	
α Serpentis,		+ 48	− 136	− 10	+ 1	
α Scorpii,		+ 41	− 131	− 10	− 9	
α Herculis,		− 5	+ 66	− 8	− 2	
α Ophiuchi,		− 27	+ 350	− 3	− 2	
α Lyræ,		+ 2	+ 86	− 26	− 1	
γ Aquilæ,		+ 69	− 194	− 10	− 1	
α Aquilæ,		+ 7	+ 35	− 20	− 1	
β Aquilæ,		+ 39	− 216	+ 5	− 1	
α^2 Capricorni,		− 6	− 6	− 12	− 2	
α Aquarii,		− 34	+ 165	− 7	+ 6	
α Pegasi,		+ 9	+ 18	− 4	+ 9	
α Andromedæ,		− 12	+ 185	0	+ 14	

We have here added the stars Sirius and Procyon to the list because Auwers has determined their irregular proper motions with sufficient accuracy to allow of their being used as standards of comparison for a long time to come. The following are the details of the operations by which Auwers' results have been applied in the above table.

PROCYON.

By his researches on the proper motion of Procyon, Auwers has found a certain correction, q, to its right ascension, defined as follows: Represent by q_1, q_2, etc., to q_6, the several corrections to be applied to the right ascension of Procyon in Wolfers' "Tabulæ Reductionum," in order that the respective differences between the corrected right ascension and the right ascensions of α Tauri, β Orionis, α Orionis, β Geminorum, α Hydræ, and α Leonis, given in the same tables, may be correct. Then Auwers puts—

$$q = \tfrac{1}{6}\left(q_1 + q_2 + q_3 + q_4 + q_5 + q_6\right)$$

It follows that if we represent by Δ_1, Δ_2, etc., to Δ_6, the definitive corrections to Wolfers' positions of these six stars, the correction to Wolfers' position of Procyon will be—

$$\tfrac{1}{6}\Sigma\Delta + q$$

But this result requires a small change arising in this way. Wolfers and Auwers have assumed that the corrections to the mean places of the Tabulæ Regiomontanæ increase uniformly with the time. In other words, it is assumed by Auwers that q is of the form—

$$x + yt + \mathrm{P}$$

P being the purely periodic terms depending on the orbital motion. But, on examining the values of the terms depending on the second and third powers of the time in the preceding table it will be seen that the correction is really of the form—

$$x + y\,t - 0^s.031\ \mathrm{T}^2 + \mathrm{P}$$

To determine rigorously the correction to q which would have been introduced had Auwers taken account of the term in T^2 would be difficult. Owing to the extreme minuteness of the correction, we assume that the effect of changing the term would correspond to the effect of a similar change in the equations used in this paper, or—

$$\Delta x = \Delta\,\mathrm{R.\,A.}\ (1820) = -.08a = +0^s.005$$
$$\Delta y = -.13a = -.13 \times -62 \quad = +0^s.008$$

The correction to Auwers will then be—

$$+3 + 8\ \mathrm{T} - 31\ \mathrm{T}^2$$

T being the fraction of a century after 1800.

The value of this correction will be only −.006 in 1755, and will not exceed $0^s.003$ during the period 1800 to 1860.

We therefore conclude that the correction to the position of Procyon given in the Tabulæ Reductionum is—

$$\Delta + 3 + 8\ \mathrm{T} - 31\ \mathrm{T}^2 + q$$

It remains to determine Δ. Subtracting Wolfers' corrections to the positions of the Tabulæ Regiomontanæ, as found in the Tabulæ Reductionum, pp. xxxv, xxxvi, from the corrections given in the preceding table, it seems that our corrections to Wolfers' positions are as follows:

α Tauri	−25	−17 T	−12 T²	−6 T³
β Orionis	+2	−61	−12	−2
α Orionis	+1	−99	−4	−7
β Geminorum	−35	+15	+2	−14
α Hydræ	−40	+28	−4	+4
α Leonis	−47	+51	−10	+6
Mean	Δ = −24	−14 T	−7 T²	−3 T³

The correction to Wolfers' position of Procyon is, therefore,

$$-21 - 6\ \mathrm{T} - 38\ \mathrm{T}^2 - 3\ \mathrm{T}^3 + q$$

Adding Wolfers' correction to Bessel's position of Procyon, which is—

$$-6 + 133\ \mathrm{T}$$

we have the result given in the table for the correction to the right ascension of Procyon in the Tabulæ Regiomontanæ.

SIRIUS.

Auwers' correction to the position of Sirius is given in a form similar to that adopted for Procyon, but the stars of comparison are β Orionis, α Orionis, and Procyon. The positions adopted for comparison are those of the Tabulæ Regiomontanæ, except that the right ascension of Procyon is first corrected by the quantity—

$$-0^s.025 - 0^s.001386\ (t - 1825) - 0^s.0705 \cos 9^\circ{:}00634\ (t - 179557)$$

or by the quantity—

$$+10 - 139\ \mathrm{T} + \mathrm{P}$$

P being the periodic term. The terms in the correction to the relative motion of Sirius which depend on the second and third powers of the time are—

$$-34\ \mathrm{T}^2 + 2\mathrm{T}^3$$

and, reasoning as in the case of Procyon, we conclude that Auwers' q should be corrected by the quantity—

$$+4 + 9\ \mathrm{T} - 34\ \mathrm{T}^2 + 2\ \mathrm{T}^3$$

The value of this correction for different dates is—

	s.
1760	—.005
1780	+.001
1800	+.004
1820	+.004
1840	+.002
1860	—.003
1880	—.010

The correction to the right ascension of Sirius in the Tabulæ Regiomontanæ is, therefore,

$$\varDelta + 4 + 9\ \mathrm{T} - 34\ \mathrm{T}^2 + 2\ \mathrm{T}^3 + q$$

$\varDelta$ being the mean correction to the right ascensions of β Orionis, α Orionis, and Procyon, the latter star being, however, first corrected by the expression given above.

Comparing this expression with Auwers' q, (A. N. No. 1382, p. 62,) the value of which is—

$$39 - 119\ \mathrm{T} - 0^s.0705 \cos n\ (t - \mathrm{T})$$

we see that the preliminary correction to the position of Procyon is—

$$q - 29 - 20\ \mathrm{T}$$

Subtracting this correction from the above correction to Procyon, taking the corrections to β Orionis and α Orionis without change, and representing by Δ_1, Δ_2, and Δ_3 the individual corrections to Auwers' assumed positions of the three stars of comparison, we have—

$$\begin{array}{lrrrr}
\Delta_1 = & +41 & +\ 27\,T & -12\,T^2 & -2\,T^3 \\
\Delta_2 = & +24 & -\ 10 & -\ 4 & -7 \\
\Delta_3 = & +\ 2 & +147 & -38 & -3 \\
\Delta\ = & +22 & +\ 55\,T & -18\,T^2 & -4\,T^3
\end{array}$$

Adding this to the expression which replaces Auwers' q, we have the result given in the above table.

§ 7.

CORRECTIONS TO DIFFERENT CATALOGUES TO REDUCE THEM TO A MEAN SYSTEM.

The preceding investigations have resulted in giving us a set of 29 right ascensions, which we may suppose to be free from all systematic errors whatever, and of which we may suppose the mean accidental error to be much less than that of any one catalogue determination. We may, therefore, suppose any systematic differences which exhibit themselves between the right ascensions of any catalogue and those just obtained to be due to errors of the former, and from the entire body of the differences we may obtain the correction necessary to reduce it to the standard. This correction may be considered to consist of three parts—

1. A constant, which we may consider as a correction to the adopted equinox.

2. Periodic terms depending on the right ascension, which in all recent catalogues we may suppose to be of the form—

$$m \cos \alpha + n \sin \alpha$$

and to which terms of double period—

$$m' \cos 2\,\alpha + n' \sin 2\,\alpha$$

may be added, if it be deemed necessary. In some of the older catalogues, especially in Bessel I, the terms of double period appear to be quite sensible, but they ought not to be real in any modern catalogue.

3. A term depending on the declination, which, within the limits in which the present investigations are applicable, may be regarded as proportional to the simple declination. In application, it might be better to suppose it proportional to the sine of the declination. The limits in question may be considered as 40° north declination and 30° south declination, but the correction in question will necessarily become uncertain as we approach either limit.

These three corrections have been obtained in the order in which they are numbered, as follows: Represent by—

$$c_1,\ c_2,\ \text{- - - - -}\ c_{29}$$

the several excesses of the standard right ascensions over those of the catalogue to be compared. The mean value of c may then be taken as the correction for equinox, which we may represent by e. Subtract e from each individual c, and represent the residuals by c'_1, c'_2, etc. The values of m and n may then be obtained in the same manner by which the periodic terms of the residuals of the separate catalogues have been obtained. But the addition of Sirius and Procyon to the list of stars will make a slight change in the equations, so that we shall have—

$$11.9\,m = \Sigma c' (\cos \alpha - .03 \sin \alpha)$$
$$17.1\,n = \Sigma c' (\sin \alpha - .04 \cos \alpha)$$

The values of the coefficients—

$$\cos \alpha - .03 \sin \alpha \text{ and } \sin \alpha - .04 \cos \alpha$$

for each star are given in the following table.

Star.	cos α	sin α	Co-efficient for—	
			11.9m	17.1n
α Andromedæ	+ 1.00	.00	+ 1.00	— .04
γ Pegasi	+ 1.00	+ .03	+ 1.00	— .01
α Arietis	+ .87	+ .50	+ .86	+ .47
α Ceti	+ .73	+ .69	+ .71	+ .66
α Tauri	+ .39	+ .92	+ .36	+ .90
β Orionis	+ .22	+ .97	+ .19	+ .96
β Tauri	+ .19	+ .98	+ .16	+ .97
α Orionis	+ .06	+ 1.00	+ .03	+ 1.00
α Canis Majoris	— .16	+ .99	— .19	+ 1.00
α Canis Minoris	— .38	+ .92	— .41	+ .93
β Geminorum	— .40	+ .91	— .43	+ .93
α Hydræ	— .76	+ .64	— .78	+ .67
α Leonis	— .86	+ .51	— .88	+ .54
β Leonis	— 1.00	+ .09	— 1.00	+ .13
α Virginis	— .95	— .32	— .94	— .28
α Bootis	— .85	— .53	— .83	— .50
α^2 Libræ	— .77	— .64	— .75	— .61
α Coronæ Borealis	— .62	— .78	— .60	— .76
α Serpentis	— .59	— .81	— .57	— .79
α Scorpii	— .43	— .90	— .40	— .88
α Herculis	— .23	— .97	— .20	— .96
α Ophiuchi	— .16	— .99	— .13	— .98
α Lyræ	+ .13	— .99	+ .16	— 1.00
γ Aquilæ	+ .41	— .91	+ .44	— .93
α Aquilæ	+ .43	— .90	+ .46	— .92
β Aquilæ	+ .45	— .89	+ .48	— .91
α^2 Capricorni	+ .53	— .85	+ .56	— .87
α Aquarii	+ .86	— .51	+ .88	— .54
α Pegasi	+ .96	— .28	+ .97	— .32

Subtracting from each c' the corresponding value of—

$$m \cos \alpha + n \sin \alpha$$

the residuals ought to be purely accidental errors, unless they are found to depend on the declination. In this case, if we suppose a term proportional to the simple declination, we may represent it in the form—

$$d \times \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

the zero of reference being 10° north for convenience, because that is nearly the mean declination of all the adopted standard stars, and the degrees of declination being divided by 10 in order to have a more convenient factor. We may then find d by solving the equations of condition formed by putting the above expression equal to c'' for each star. The following is a convenient and sufficiently accurate expression for d:

$60\,d = +2\,c''$ of α Andromedæ.
$+1$ . . α Arietis.
-1 . . α Ceti.
$+1$ . . α Tauri.
-2 . . β Orionis.
$+2$ . . β Tauri.
-2 . . α Canis Majoris.
$+2$ . . β Geminorum.
-2 . . α Hydræ.
$+1$ . . β Leonis.
-2 . . α Virginis.
$+1$ . . α Bootis.
-2 . . α^2 Libræ.
$+2$ . . α Coronæ.
-2 . . α Scorpii.
$+1$ . . α Herculis.
$+2$ . . α Lyræ.
-2 . . α^2 Capricorni.
-1 . . Aquarii.

As an example of the method we may discuss the twelve-year series of Paris observations, extending from 1856 to 1867. The different results are shown in the following table. First we give the mean correction to Le Verrier's fundamental catalogue deduced from all the observations, using the mean results given in each volume. Next we have the resulting corrections to Bessel. Subtracting the correction to Bessel for the mean epoch, we have the correction to the standard of this paper, or $-c$. The mean value of c is found to be 32, which is the value of e. Adding this, we have $-c'$.

From the values of c' we find—

$$m = +20$$
$$n = -23$$

Adding $20 \cos \alpha - 23 \sin \alpha$, we have c''. From c'' we have—

$$d = +9$$

Applying $9 \dfrac{\text{Dec.} - 10^\circ}{10^\circ}$ to $-c''$ we have r, or the definitive corrections to the standard given by the Paris observations.

Star.	Corrections to—		$-c$	$-c'$	Computed periodic terms.	$-c''$	r
	Le Verrier.	Tab. Reg.					
α Andromedæ . .	− 15	+ 57	− 47	− 15	+ 20	+ 5	+ 21
γ Pegasi . . .	− 16	− 127	− 69	− 37	+ 20	− 17	− 13
α Arietis . . .	− 31	+ 33	− 57	− 25	+ 5	− 20	− 8
α Ceti	+ 34	+ 53	− 27	+ 5	− 1	+ 4	− 2
α Tauri	− 25	+ 49	+ 3	+ 35	− 13	+ 22	+ 27
β Orionis . . .	+ 52	+ 76	+ 23	+ 55	− 18	+ 37	+ 21
β Tauri	− 6	− 59	− 18	+ 14	− 19	− 5	+ 11
α Orionis . . .	+ 16	+ 13	− 3	+ 29	− 22	+ 7	+ 4
α Canis Majoris .	(− 228)	+ 39	− 4	+ 28	− 26	+ 2	− 22
α Canis Minoris .	(+ 51)	+ 37	− 4	+ 28	− 29	− 1	− 6
β Geminorum . .	− 6	+ 92	− 24	+ 8	− 29	− 21	− 5
α Hydræ . . .	+ 61	+ 157	+ 12	+ 44	− 30	+ 14	− 2
α Leonis . . .	+ 20	+ 98	0	+ 32	− 29	+ 3	+ 6
β Leonis . . .	+ 4	+ 86	− 32	0	− 22	− 22	− 17
α Virginis . . .	+ 28	0	− 24	+ 8	− 11	− 3	− 21
α Bootis . . .	+ 7	+ 30	− 39	− 7	− 4	− 11	− 2
α^2 Libræ	+ 23	− 66	+ 1	+ 33	0	+ 33	+ 10
α Corona . . .	− 25	+ 18	− 60	− 28	+ 6	− 22	− 7
α Serpentis . . .	+ 28	− 74	− 36	− 4	+ 7	+ 3	0
α Scorpii . . .	+ 46	− 36	+ 6	+ 38	+ 12	+ 50	+ 18
α Herculis . . .	− 11	− 27	− 59	− 27	+ 18	− 9	− 4
α Ophiuchi . . .	− 1	+ 146	− 37	− 5	+ 20	+ 15	+ 18
α Lyræ	− 63	− 48	− 93	− 61	+ 25	− 36	− 10
γ Aquilæ . . .	− 5	− 113	− 63	− 31	+ 29	− 2	− 2
α Aquilæ . . .	− 24	− 44	− 65	− 33	+ 29	− 4	− 5
β Aquilæ . . .	+ 6	− 148	− 60	− 28	+ 29	+ 1	− 3
α^2 Capricorni . .	+ 36	− 49	− 33	− 1	+ 30	+ 29	+ 8
α Aquarii . . .	+ 10	+ 9	− 55	− 23	+ 29	+ 6	− 4
α Pegasi . . .	− 42	− 42	− 62	− 30	+ 25	− 5	− 1

The systematic corrections obtained in this way for a number of catalogues are given in the following list. The year attached to each catalogue is that to which it is reduced. Systematic corrections much smaller than the accidental errors of the catalogue may generally be omitted entirely.

SYSTEMATIC CORRECTIONS TO BE APPLIED TO THE RIGHT ASCENSIONS OF EQUATORIAL AND ZODIACAL STARS IN DIFFERENT CATALOGUES TO REDUCE THEM TO A MEAN SYSTEM.

AUWERS' BRADLEY, (1755.)*

$$\varDelta = -0^s.010 + 0^s.003 \cos\alpha - 0^s.007 \sin\alpha$$

The mean value of the periodic terms being less than one-third the probable error of the individual star-places, they may be neglected entirely.

BESSEL'S FUNDAMENTA, 1755.

$$\varDelta = -0^s.01$$

DE ZACH'S FIXARUM PRÆCIPUARUM CATALOGUS NOVUS EX OBSERVATIONIBUS ASTRONOMICIS IN SPECULA ASTRONOMICA GOTHANA ANNIS 1787, 1788, 1789, 1790, HABITIS ET AD INITIUM ANNI MDCCC. CONSTRUCTIS.

$$\varDelta = +0^s.36 - 0^s.03 \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

PIAZZI, 1800.

$$\varDelta = +0^s.10 + 0^s.028 \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

MASKELYNE, (1805.)

Catalogue in vol. 4 of his observations, second part, p. 112.

$$\varDelta = +0^s.145 + 0^s.019 \cos\alpha + 0^s.006 \sin\alpha - 0^s.012 \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

AUWERS' CACCIOTORE, (1805.)

$$\varDelta = -0^s.006 - 0^s.010 \cos\alpha - 0^s.033 \sin\alpha$$

BESSEL'S FIRST FUNDAMENTAL CATALOGUE, (1815.)

$$\varDelta = -0^s.037 - 0^s.037 \cos\alpha + 0^s.037 \sin\alpha$$

BESSEL'S SECOND FUNDAMENTAL CATALOGUE, (1825.)

$$\varDelta = +0^s.020 - 0^s.006 \cos\alpha + 0^s.010 \sin\alpha$$

STRUVE, (1825.)

$$\varDelta = -0^s.020 + 0^s.007 \cos\alpha + 0^s.005 \sin\alpha + 0^s.008 \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

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*Auwers' catalogue being still unpublished, this correction must be understood as applicable only to the preliminary catalogue, in which no correction to Bessel's equinox has been determined or applied.

BRINKLEY, (1824.)

$$\varDelta = +0^s.150$$

ARGELANDER, (1830.)

$$\varDelta = +0^s.017 - 0^s.006\,\frac{\text{Dec.} - 10^\circ}{10}$$

GILLISS'S FIRST WASHINGTON CATALOGUE, 1840.

(*From observations made at the old Naval Observatory during the years* 1838–'42. *Washington,* 1846. 8vo.)

$$\varDelta = +0^s.032 \cos\alpha - 0^s.028 \sin\alpha$$

POND, 1830.

$$\varDelta = -0^s.056 - 0^s.015 \sin\alpha$$

(For stars in south declination the additional positive correction $0^s.0015 \times$ south declination in degrees is strongly indicated.)

AIRY'S CAMBRIDGE CATALOGUE.

In the Memoirs of the Royal Astronomical Society, vol. XI.

$$\varDelta = -0^s.024 - 0^s.018 \sin 2\alpha$$

(The term of double period, though *a priori* somewhat improbable, is strongly indicated by the comparisons.)

TAYLOR, (1835.)

General catalogue of the principal fixed stars from observations made at the honorable the East India Company's observatory at Madras in the years 1830–1843.

$$\varDelta = -0^s.047 + 0^s.023 \cos\alpha + 0^s.0011\,\frac{\text{Dec.} - 10^\circ}{10^\circ}$$

The last term is far within the probable error of the catalogue-positions.

GREENWICH 12-YEAR CATALOGUE.

Catalogue for 1840.

$$\varDelta = +0^s.067 + 0^s.014 \cos\alpha - 0^s.034 \sin\alpha$$

Catalogue for 1845.

$$\varDelta = +0^s.019 + 0^s.007 \cos\alpha - 0^s.025 \sin\alpha$$

Robinson's Armagh Catalogue, (1840.)

$$\Delta = +0^s.020.$$

(No positions of fundamental stars are given in this catalogue. The correction has, therefore, been derived from a comparison of all the stars south of 45° north polar distance which are common to Armagh and the Greenwich 6-Year Catalogue for 1840, and observed at least five times at each observatory. The differences between these catalogues were corrected for the error of the Greenwich catalogue, and the outstanding differences were presumed to be due to error of the Armagh catalogue.)

Pulkowa, (1845.)

$$\Delta = +0^s.019$$

Radcliffe Catalogue, 1845.

$$\Delta = -0^s.009 + 0^s.047 \cos \alpha - 0^s.020 \sin \alpha$$

(The positions of the fundamental stars in this catalogue give the correction—

$$\Delta = +0^s.001 + 0^s.055 \cos \alpha - 0^s.021 \sin \alpha$$

In the introduction to the catalogue, Mr. Main gives a comparison of this catalogue with Greenwich, 1845, from which I derive—

$$\text{Greenwich} - \text{Radcliffe} = -0^s.038 + 0^s.032 \cos \alpha + 0^s.005 \sin \alpha$$

Adding the Greenwich correction, this will give, for Radcliffe,

$$\Delta = -0^s.019 + 0^s.039 \cos \alpha - 0^s.020 \sin \alpha$$

The mean result has been adopted.)

Greenwich 6-Year Catalogue, (1850.)

$$\Delta = -0^s.003 + 0^s.011 \cos \alpha - 0^s.023 \sin \alpha$$

Greenwich 7-Year Catalogue, (1860.)

$$\Delta = +0^s.014 + 0^s.014 \cos \alpha - 0^s.012 \sin \alpha$$

Greenwich 7-Year Catalogue, (1864.)

$$\Delta = +0^s.033 + 0^s.010 \cos \alpha - 0^s.010 \sin \alpha$$

Paris Annual Catalogues, 1856–'67.

$$\Delta = +0^s.032 + 0^s.020 \cos \alpha - 0^s.023 \sin \alpha + 0^s.009 \frac{\text{Dec.} - 10^\circ}{10^\circ}$$

Washington "Positions of Fundamental Stars," (1870.)

(In Appendix III to the Washington Observations for 1867.)

$$\Delta = -0^s.024$$

Washington Annual Results of Observations with the Old Transit-Instrument, (1862–'65.)

$$\varDelta + = 0^s.006 + 0^s.021 \cos \alpha - 0^s.024 \sin \alpha$$

Washington Annual Results of Observations with the Transit Circle, (1866–'72.)

$$\varDelta = + 0^s.006$$

Those of the above corrections which form a considerable part of the probable error of the catalogue-positions are tabulated in the following tables. In the corrections depending on the declinations the constant term is included, except for Paris, where it is included in the first table.

Corrections to different catalogues depending on the right ascension.

R. A.	Airy's Cambridge catalogue.	Taylor's Madras catalogue.	AIRY'S GREENWICH CATALOGUES.					Paris annual catalogues, 1855–'67.	Radcliffe catalogue, 1845.
			1840.	1845.	1850.	1860.	1864.		
h.	s.	s.	s.	s.	s.	s.	s.	s.	s.
0	− .02	− .02	+.081	+.026	+.008	+.028	+.043	+ .052	+ .04
1	− .03	− .02	+.072	+.020	+.002	+.025	+.040	+ .045	+ .03
2	− .04	− .03	+.062	+.013	−.005	+.021	+.037	+ .037	+ .02
3	− .04	− .03	+.053	+.006	−.011	+.015	+.033	+ .030	+ .01
4	− .04	− .04	+.045	+.001	−.017	+.010	+.029	+ .022	− .01
5	− .03	− .04	+.038	−.003	−.022	+.005	+.026	+ .015	− .02
6	− .02	− .05	+.033	−.006	−.026	+.002	+.023	+ .009	− .03
7	− .02	− .05	+.030	−.007	−.028	.0	+.021	+ .005	− .04
8	− .01	− .06	+.031	−.007	−.028	−.003	+.019	+ .002	− .05
9	− .01	− .06	+.033	−.004	−.027	−.004	+.019	+ .002	− .06
10	− .01	− .07	+.038	+.001	−.024	−.004	+.019	+ .003	− .06
11	− .02	− .07	+.044	+.006	−.020	−.003	+.021	+ .007	− .06
12	− .02	− .07	+.053	+.012	−.014	.0	+.023	+ .012	− .06
13	− .03	− .07	+.062	+.018	−.008	+.003	+.026	+ .019	− .05
14	− .04	− .07	+.072	+.025	−.001	+.007	+.029	+ .027	− .04
15	− .04	− .06	+.081	+.032	+.005	+.013	+.033	+ .034	− .03
16	− .04	− .06	+.089	+.037	+.016	+.018	+.037	+ .042	− .02
17	− .03	− .05	+.096	+.041	+.016	+.023	+.040	+ .049	.00
18	− .02	− .05	+.101	+.044	+.020	+.026	+.043	+ .055	+ .01
19	− .02	− .04	+.104	+.045	+.022	+.028	+.045	+ .059	+ .02
20	− .01	− .04	+.103	+.045	+.022	+.031	+.047	+ .062	+ .03
21	− .01	− .03	+.101	+.042	+.021	+.032	+.047	+ .062	+ .04
22	− .01	− .03	+.096	+.037	+.018	+.032	+.047	+ .061	+ .04
23	− .02	− .02	+.090	+.032	+.014	+.031	+.045	+ .057	+ .04
24	− .02	− .02	+.081	+.026	+.008	+.028	+.043	+ .052	+ .04

Corrections depending on the declination.

Dec.	N. P. D.	Piazzi, 1800.	Struve, Posit. Med.	Argelander, 1830.	Paris annual catalogues, 1856–'63.
°	°	s.	s.	s.	s.
+40	50	+ .18	+ .00	− .001	+ .027
35	55	+ .17	.000	+ .002	+ .023
30	60	+ .16	− .004	+ .005	+ .018
25	65	+ .14	− .008	+ .008	+ .014
20	70	+ .13	− .012	+ .011	+ .009
15	75	+ .11	− .016	+ .014	+ .004
10	80	+ .10	− .020	+ .017	.0
+ 5	85	+ .09	− .024	+ .020	− .004
0	90	+ .07	− .028	+ .023	− .009
− 5	95	+ .06	− .032	+ .021	− .014
−10	100	+ .04	− .036	+ .029	− .018
−15	105	+ .03	− .040	+ .032	− .022
−20	110	+ .02	− .044	+ .035	− .027
−25	115	.00	− .048	+ .038	− .032

§ 8.

DEFINITIVE RIGHT ASCENSIONS OF THE FUNDAMENTAL STARS.

Applying these corrections to the catalogue-positions, we may deduce definitive corrections to the individual right ascensions of the stars under consideration. If, in doing this, we employed only the catalogues used in the first determination, and assigned the same relative weights as in the first discussion, we should reproduce the right ascensions already found. But the weights were first assigned solely with respect to the probable freedom of the catalogue from periodic errors dependent on the right ascension, while, after elimination of these errors, they should be assigned with respect to the purely accidental errors of the determinations, and this change of weights will make a small change in the individual definitive positions. We can also add a number of catalogues not employed in the preliminary determination, particularly the Greenwich and Paris catalogues, which will add considerably to the precision of the results.

The details of the process are shown in the following tables. Under each star is given, in the first column, the difference of excess of catalogue-place, corrected for systematic error over the standard place already deduced.

In two cases several catalogues of small weight have been combined, as the exhibition of the individual results of each catalogue would not be of interest. The first set are the catalogues of—

Piazzi - - - - - - -	1800, weight 1
Maskelyne - - - -	1805, weight 2
Auwers' Cacciotore - -	1805, weight 2
Bessel - - - - - - -	1815, weight 1

which have been combined into a single result for the mean date 1807. The mean systematic correction was so small that only the correction for equinox was actually applied. The next case was Airy's Cambridge catalogue and Pond's general catalogue, which were combined into a single result.

No corrections depending on the declination have been applied, except in the case of Paris and Argelander.

Under the titles Greenwich, 1869, and Washington, 1869, are given the mean results of the Greenwich and Washington observations for the years 1868, 1869, and 1870, without any correction for periodic error.

The weights assigned are constant for each catalogue, except in the case of Bradley, where the weight 2 was assigned to results depending on more than 100 observations, and 1 to those depending on less. The adopted weights and general solution are as follows, the time T being reckoned from 1845:

No.	Catalogue.	Date.	Weight.	T'	C'_x	C''_x	C'_y	C''_y
1	Bradley	1755	1 or 2	— .90	.035	.054	— .46	— .63
2	Combined catalogue .	1807	1	— .38	.029	.024	— .20	— .14
3	Bessel II	1823	2	— .22	.055	.048	— .24	— .16
4	Struve.	1824	1	— .21	.027	.023	— .11	— .07
5	Argelander	1828	4	— .17	.108	.095	— .39	— .24
6	Pond and Airy . . .	1830	1	— .15	.026	.023	— .08	— .05
7	Greenwich	1839	2	— .06	.050	.048	— .08	— .05
8	Greenwich	1845	2	.0	.049	.048	— .02	.0
9	Pulkowa	1845	4	.0	.098	.095	— .05	.0
10	Greenwich	1851	3	+ .06	.072	.071	+ .05	+ .06
11	Greenwich	1857	3	+ .12	.070	.071	+ .14	+ .12
12	Paris	1862	4	+ .17	.092	.095	+ .25	+ .21
13	Greenwich	1864	4	+ .19	.090	.095	+ .33	+ .26
14	Washington	1865	5	+ .20	.112	.118	+ .44	+ .35
15	Greenwich	1869	2	+ .24	.044	.047	+ .21	+ .17
16	Washington	1869	2	+ .24	.044	.047	+ .21	+ .17

The equations given by the several catalogues, when solved by least squares, lead to the result that—

$$\Delta \text{ R. A. for } 1845 = \Sigma\, C_x \times r$$
$$\Delta \text{ Ann. variation} = \Sigma\, C_y \times r$$

r representing the correction given by any catalogue, and C_x, C_y, factors constant for each catalogue, except that there will be two systems of these factors, the one corresponding to the stars for which the results of Bradley's observations receive the weight 1, the other to those in which they receive the weight 2. The former are designated by one accent, the latter by two. In the case of C_x the differences are so minute that a single mean value has been actually used.

In the column r', under each star, is given the outstanding residual after subtracting the concluded correction—

$$\Delta x + T' \Delta y$$

Sirius and Procyon are included only for the sake of comparison. No correction to Auwers' places is indicated.

Residual corrections given by catalogues.

No.	Catalogue.	Year.	α Androm.		γ Pegasi.		α Arietis.		α Ceti.		α Tauri.		β Orionis.	
			r	*r'*	*r*	*r'*	*r*	*r'*	*r*	*r'*	*r*	*r'*	*r*	*r'*
1	Bradley	1755	+ 15	+ 12	− 29	− 49	+ 9	+ 4	+ 32	+ 45	+ 24	+ 17	+ 3	+ 6
2	Combined catalogue	1807	+ 9	+ 4	+ 23	+ 17	− 12	− 11	− 24	− 20	− 6	− 18	− 7	− 4
3	Bessel II	1823	− 12	− 18	+ 27	+ 25	− 12	− 10	− 30	− 29	+ 5	− 8	− 25	− 22
4	Struve	1824	− 61	− 67	− 1	− 3	+ 2	+ 5	+ 1	+ 2	− 15	− 28	+ 53	+ 56
5	Argelander	1828	+ 1	− 5	+ 15	+ 14	− 8	− 5	− 10	− 10	+ 2	− 11	− 10	− 7
6	Pond and Airy	1830	+ 27	+ 20	+ 40	+ 40	+ 2	+ 5	− 28	− 28	+ 22	+ 8	− 11	− 8
7	Greenwich	1839	+ 26	+ 19	0	+ 2	− 28	− 24	+ 40	+ 38	+ 42	+ 28	− 13	− 10
8	Greenwich	1845	+ 6	− 1	− 14	− 11	+ 23	+ 28	− 23	− 26	+ 44	+ 29	− 3	0
9	Pulkowa	1845	+ 8	+ 1	− 3	0	+ 14	+ 19	+ 4	+ 1	− 10	− 25	+ 6	+ 9
10	Greenwich	1851	+ 23	+ 15	− 7	− 2	0	+ 6	0	− 4	+ 21	+ 6	− 2	0
11	Greenwich	1857	+ 28	+ 20	− 2	+ 4	− 9	− 3	− 14	− 19	+ 18	+ 2	− 26	− 24
12	Paris	1862	+ 21	+ 13	− 13	− 5	− 8	− 1	− 2	− 7	+ 27	+ 11	+ 21	+ 23
13	Greenwich	1864	+ 9	+ 1	− 18	− 10	− 6	+ 1	+ 8	+ 2	+ 19	+ 3	− 4	− 2
14	Washington	1865	+ 1	− 7	− 13	− 4	− 7	0	+ 21	+ 15	+ 14	− 3	− 4	− 2
15	Greenwich	1869	+ 2	− 6	− 21	− 11	− 30	− 23	− 10	− 16	+ 26	+ 9	− 14	− 12
16	Washington	1869	− 22	− 30	− 1	+ 9	− 10	− 3	+ 43	+ 37	+ 11	− 6	+ 6	+ 8

No.	Catalogue.	Year.	β Tauri.		α Orionis.		α Canis Maj.		α Canis Min.		β Gemin.		α Hydræ.	
1	Bradley	1755	+ 4	+ 2	− 9	− 9	0	.	0	.	+ 11	+ 7	− 5	− 12
2	Combined catalogue	1807	+ 32	+ 25	+ 27	+ 26	.	.	.	.	− 4	− 6	− 2	− 4
3	Bessel II	1823	− 4	− 12	+ 2	0	.	.	.	.	− 12	− 13	− 23	− 24
4	Struve	1824	− 22	− 30	− 14	− 16	.	.	.	.	− 23	− 24	+ 25	+ 24
5	Argelander	1828	+ 3	− 6	+ 14	+ 12	.	− 17	.	+ 8	0	− 1	+ 10	+ 10
6	Pond and Airy	1830	+ 15	+ 6	+ 32	+ 30	.	+ 5	.	+ 14	+ 10	+ 9	+ 48	+ 48
7	Greenwich	1839	− 4	− 14	+ 4	+ 2	.	+ 2	.	− 24	+ 11	+ 11	− 15	− 14
8	Greenwich	1845	+ 6	− 4	− 5	− 7	.	− 6	.	+ 13	+ 3	+ 3	+ 3	+ 4
9	Pulkowa	1845	+ 14	+ 4	− 13	− 15	.	.	.	.	+ 6	+ 6	+ 5	+ 6
10	Greenwich	1851	+ 17	+ 6	− 5	− 8	.	+ 4	.	+ 2	− 3	− 3	− 6	− 4
11	Greenwich	1857	+ 34	+ 23	+ 3	0	.	+ 12	.	− 12	+ 7	+ 7	− 4	− 1
12	Paris	1862	+ 11	− 1	+ 4	+ 1	.	− 11	.	− 6	− 5	− 5	− 2	+ 1
13	Greenwich	1864	+ 16	+ 4	+ 16	+ 13	.	− 17	.	+ 7	+ 6	+ 7	− 8	− 5
14	Washington	1865	− 14	− 26	+ 1	− 2	.	+ 16	.	0	− 11	− 10	− 12	− 9
15	Greenwich	1869	+ 36	+ 24	+ 2	− 1	.	− 3	.	+ 23	+ 9	+ 10	+ 26	+ 30
16	Washington	1869	+ 19	+ 7	+ 1	− 2	.	+ 1	.	+ 15	− 7	− 6	− 23	− 19

Residual corrections given by catalogues.

No.	Catalogue.	Year.	α Leonis.		β Leonis.		α Virginis.		α Bootis.		α^2 Libræ.		α Coronæ.	
			r	r'	r	r'	r	r'	r	'	r	r'	r	r'
1	Bradley	1755	+ 12	+ 6	+ 1	+ 11	− 5	− 13	− 1	− 6	− 7	− 8	+ 5	− 11
2	Combined catalogue	1807	− 2	− 8	+ 10	+ 17	− 2	0	+ 5	+ 3	+ 3	− 1	− 28	− 35
3	Bessel II	1823	+ 1	− 5	− 26	− 20	+ 22	+ 27	− 3	− 3	− 3	− 8	− 1	− 5
4	Struve	1824	− 4	− 10	− 12	− 7	− 16	− 10	− 17	− 17	− 19	− 24	+ 30	+ 26
5	Argelander . . .	1828	− 8	− 14	− 2	+ 3	0	+ 6	− 9	− 9	+ 19	+ 13	+ 9	+ 6
6	Pond and Airy . .	1830	− 9	− 15	− 3	+ 2	− 15	− 8	− 6	− 6	+ 26	+ 20	+ 16	+ 13
7	Greenwich	1839	+ 38	+ 32	− 10	− 6	− 15	− 6	+ 24	+ 25	+ 28	+ 23	− 5	− 6
8	Greenwich	1845	+ 11	+ 5	− 30	− 26	− 15	− 5	+ 6	+ 7	+ 30	+ 24	+ 9	+ 9
9	Pulkowa	1845	+ 2	− 4	+ 6	+ 10	+ 8	+ 18	+ 10	+ 11	− 11	− 17	− 2	− 2
10	Greenwich	1851	+ 6	+ 1	+ 4	+ 8	− 16	− 5	+ 15	+ 16	+ 4	− 2	+ 28	+ 28
11	Greenwich	1857	+ 16	+ 11	− 1	+ 2	− 36	− 24	+ 8	+ 10	− 9	− 15	+ 15	+ 17
12	Paris	1862	+ 6	+ 1	− 17	− 14	− 21	− 8	− 2	0	+ 10	+ 3	− 7	− 5
13	Greenwich	1864	− 5	− 10	+ 5	+ 8	− 22	− 9	− 6	− 4	+ 16	+ 9	0	+ 3
14	Washington . . .	1865	+ 5	0	− 3	0	+ 1	+ 15	− 12	− 10	0	− 7	− 18	− 15
15	Greenwich	1869	+ 11	+ 6	+ 12	+ 14	− 21	− 7	− 19	− 16	+ 10	+ 3	− 10	− 6
16	Washington . . .	1869	+ 6	+ 1	− 5	− 3	− 14	0	− 10	− 7	+ 20	+ 13	− 25	− 21

No.	Catalogue.	Year.	α Serpentis.		α Scorpii.		α Herculis.		α Ophiuchi.		α Lyræ.		γ Aquilæ.	
1	Bradley	1755	+ 5	+ 1	− 25	− 8	+ 11	+ 10	+ 14	+ 16	+ 2	0	− 1	− 4
2	Combined catalogue	1807	− 44	− 46	+ 27	+ 36	− 38	− 40	− 19	− 23	+ 14	+ 14	− 2	− 4
3	Bessel II	1823	+ 6	+ 5	+ 12	+ 18	+ 12	+ 9	− 32	− 37	+ 3	+ 4	− 1	− 2
4	Struve	1824	+ 38	+ 37	+ 16	+ 22	+ 38	+ 35	+ 40	+ 35	− 45	− 44	+ 10	+ 9
5	Argelander . . .	1828	+ 11	+ 10	− 13	− 7	+ 6	+ 3	+ 10	+ 4	− 21	− 20	+ 14	+ 13
6	Pond and Airy . .	1830	+ 23	+ 23	− 35	− 30	− 29	− 32	+ 3	− 3	− 7	− 6	− 20	− 21
7	Greenwich	1839	− 24	− 24	+ 1	+ 5	− 4	− 7	+ 18	+ 11	+ 3	+ 5	+ 3	+ 2
8	Greenwich	1845	+ 1	+ 1	+ 28	+ 31	− 18	− 21	+ 12	+ 5	0	+ 2	− 5	− 5
9	Pulkowa	1845	− 3	− 3	− 15	− 12	− 19	− 22	− 17	− 24	+ 18	+ 21	+ 8	+ 8
10	Greenwich	1851	0	0	− 37	− 35	+ 22	+ 18	+ 8	0	+ 21	+ 24	− 18	− 18
11	Greenwich	1857	− 4	− 3	− 10	− 8	+ 33	+ 29	+ 24	+ 15	+ 17	+ 20	+ 10	+ 10
12	Paris	1862	0	+ 1	+ 18	+ 19	− 4	− 8	+ 18	+ 9	− 10	− 7	− 2	− 2
13	Greenwich	1864	+ 1	+ 2	− 5	− 5	+ 18	+ 14	+ 22	+ 13	+ 6	+ 9	− 12	− 12
14	Washington . . .	1865	− 4	− 3	+ 16	+ 16	+ 1	− 3	+ 6	− 4	− 5	− 2	+ 5	+ 5
15	Greenwich	1869	− 6	− 5	− 13	− 13	+ 10	+ 6	+ 10	0	− 26	− 23	− 18	− 18
16	Washington . . .	1869	+ 7	+ 8	− 18	− 18	− 3	− 7	− 2	− 12	− 34	− 31	+ 19	+ 19

Residual corrections given by catalogues.

No.	Catalogue.	Year.	α Aquilæ.		β Aquilæ.		α^2 Capricorni.		α Aquarii.		α Pegasi.			
			r	r'	r	r'	r	r'	r	r'	r	r'		
1	Bradley	1755	− 9	− 11	− 7	− 9	− 21	− 6	− 16	− 5	− 2	− 2		
2	Combined catalogue	1807	+ 15	+ 14	− 8	− 10	+ 10	+ 19	+ 4	+ 10	+ 20	+ 19		
3	Bessel II	1823	+ 9	+ 9	+ 31	+ 29	+ 31	+ 38	+ 18	+ 23	+ 7	+ 5		
4	Struve	1824	− 6	− 6	− 3	+ 1	+ 27	+ 34	− 3	+ 2	− 29	− 31		
5	Argelander . . .	1828	− 3	− 3	+ 11	+ 9	− 17	− 10	− 17	− 12	+ 7	+ 5		
6	Pond and Airy . .	1830	− 7	− 7	− 32	− 34	− 59	− 53	+ 4	+ 8	− 2	− 4		
7	Greenwich	1839	+ 23	+ 23	− 7	− 9	− 27	− 22	− 4	0	+ 10	+ 8		
8	Greenwich	1845	− 5	− 5	+ 5	+ 3	+ 5	+ 10	− 13	− 10	+ 2	0		
9	Pulkowa	1845	+ 7	+ 7	+ 1	− 1	− 1	+ 4	− 4	− 1	− 9	− 11		
10	Greenwich	1851	+ 2	+ 3	+ 12	+ 9	− 28	− 24	− 2	+ 1	+ 4	+ 1		
11	Greenwich	1857	0	+ 1	− 10	− 13	− 19	− 16	− 18	− 16	− 9	− 12		
12	Paris	1862	− 5	− 4	− 3	− 6	+ 8	+ 11	− 4	− 2	− 1	− 4		
13	Greenwich	1864	− 7	− 6	− 4	− 7	+ 10	+ 12	− 18	− 16	+ 4	+ 1		
14	Washington . . .	1865	+ 4	+ 5	− 3	− 6	− 5	− 3	+ 11	+ 13	+ 13	+ 10		
15	Greenwich	1869	− 8	− 7	+ 24	+ 21	+ 10	+ 12	− 3	− 2	− 4	− 7		
16	Washington . . .	1869	− 19	− 18	+ 7	+ 4	− 7	− 5	+ 25	+ 26	+ 19	+ 16		

There are still three of Maskelyne's fundamental stars which it is desirable, for the sake of completeness, to add, namely, α Aurigæ, α Cygni, and α Piscis Australis. Although these stars lie a little outside the limits we have set, they are frequently used as clock-stars. The corrections to the positions of the Tabulæ Regiomontanæ were obtained on the same general system with that adopted for the other stars. The tabular positions were corrected for the terms depending on the second and third powers of the time, as given in the following tables, and the catalogue-positions for systematic error given by the formulæ of the last section. From the differences the correction proportional to the time was deduced.

From the residuals r we derive the following definitive corrections to the right ascensions for 1800 and to the annual variations of the first standard:

Definitive corrections to right ascensions of Tabulæ Regiomontanæ.

Star.	Δ R. A.	Δy	Definitive correction.				
α Andromedæ	+ 5	+ 5	− 7	+190 T	+ 0 T^2	+14 T^3	
γ Pegasi	+ 8	− 26	+34	−164	− 6	+10	
α Arietis	0	− 11	− 3	+146	− 6	+ 7	
α Ceti	− 5	+ 17	+28	+ 98	− 6	+ 2	
α Tauri	+ 11	+ 8	+ 8	+ 98	−12	− 6	
α Aurigæ	.	.	− 2	+ 26	−31	−34	
β Orionis	− 3	+ 1	+38	+ 28	−12	− 2	
β Tauri	+ 6	+ 9	+72	−156	−14	−16	
α Orionis	+ 1	+ 3	+25	− 7	− 4	− 7	
α Canis Majoris	.	.	+155	+ 29	−52	− 2	+ P
α Canis Minoris	.	.	− 15	+ 8	−38	− 3	+ P
β Geminorum	+ 2	− 4	−18	+227	+ 2	−11	
α Hydræ	+ 3	− 10	−17	+266	− 4	+ 4	
α Leonis	+ 6	− 1	−23	+215	−10	+ 6	
β Leonis	− 7	+ 7	− 7	+200	+ 1	+10	
α Virginis	− 1	− 19	−17	+ 48	− 4	+ 8	
α Bootis	+ 2	− 7	− 5	+ 92	+41	+ 7	
α^2 Libræ	+ 4	+ 6	+84	−235	− 6	+ 3	
α Coronæ	+ 8	− 17	− 4	+135	− 6	+ 3	
α Serpentis	+ 2	− 5	+50	−141	−10	+ 1	
α Scorpii	− 10	+ 15	+31	−116	−10	− 9	
α Herculis	+ 2	+ 3	− 3	+ 69	− 8	− 2	
α Ophiuchi	+ 3	+ 10	−24	+360	− 3	− 2	
α Lyræ	0	− 4	+ 2	+ 82	−26	− 1	
γ Aquilæ	+ 2	− 3	+71	−197	−10	− 1	
α Aquilæ	+ 1	− 3	+ 8	+ 32	−20	− 1	
β Aquilæ	+ 2	+ 1	+41	−215	+ 5	− 1	
α^2 Capricorni	− 10	+ 12	−16	+ 6	−12	− 3	
α Cygni	.	.	−32	+184	−17	+ 3	
α Aquarii	− 7	+ 8	−41	+173	− 7	+ 6	
α Piscis Australis	.	.	−50	−152	− 6	+11	
α Pegasi	+ 1	+ 3	+10	+ 21	− 4	+ 9	

In the above table P signifies the periodic part of Auwers' correction, which is more convenient to use than q.

The following table gives the definitive right ascensions for every five years from 1750 to 1900.

I conceive that the mean error of these right ascensions, after correction for equinox, will not exceed $0^s.010$ at any time during the nineteenth century.

Mean right ascensions for beginning of each fifth Besselian fictitious year, 1750–1850.

Year.	α Andromedæ.		γ Pegasi.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	s.	h. m. s.	s.
1750	23 55 31.079	3.0665	0 0 23.520	3.0702
1755	46.413	3.0674	38.872	3.0707
1760	23 56 1.752	3.0683	54.227	3.0712
1765	17.095	3.0691	0 1 9.584	3.0716
1770	32.443	3.0700	24.943	3.0721
1775	47.795	3.0709	40.305	3.0726
1780	23 57 3.151	3.0717	55.669	3.0730
1785	18.512	3.0726	0 2 11.035	3.0735
1790	33.876	3.0735	26.404	3.0740
1795	49.246	3.0744	41.775	3.0745
1800	23 58 4.620	3.0753	57.148	3.0749
1805	19.998	3.0761	0 3 12.524	3.0754
1810	35.381	3.0770	27.902	3.0759
1815	50.768	3.0779	43.283	3.0764
1820	23 59 6.160	3.0788	58.666	3.0769
1825	21.556	3.0797	0 4 14.052	3.0774
1830	36.956	3.0806	29.440	3.0778
1835	52.361	3.0815	44.830	3.0783
1840	0 0 7.771	3.0824	0 5 0.223	3.0788
1845	23.185	3.0833	15.618	3.0793
1850	38.603	3.0842	31.016	3.0798
1855	54.026	3.0851	46.416	3.0803
1860	0 1 9.454	3.0860	0 6 1.819	3.0808
1865	24.886	3.0869	17.224	3.0813
1870	40.323	3.0879	32.632	3.0818
1875	55.764	3.0888	48.042	3.0823
1880	0 2 11.210	3.0897	0 7 3.455	3.0828
1885	26.661	3.0906	18.870	3.0833
1890	42.116	3.0915	34.288	3.0838
1895	57.575	3.0925	49.708	3.0843
1900	0 3 13.040	3.0934	0 8 5.131	3.0848

Mean right ascensions, &c.—Continued.

Year.	α ARIETIS.		α CETI.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	s.	h. m. s.	s.
1750	1 53 8.365	3.3430	2 49 14.423	3.1170
1755	25.083	3.3440	30.009	3.1175
1760	41.805	3.3450	45.598	3.1179
1765	58.533	3.3460	2 50 1.189	3.1184
1770	1 54 15.266	3.3470	16.782	3.1189
1775	32.003	3.3480	32.377	3.1194
1780	48.746	3.3490	47.975	3.1198
1785	1 55 5.493	3.3500	2 51 3.576	3.1203
1790	22.245	3.3510	19.179	3.1208
1795	39.003	3.3520	34.784	3.1213
1800	55.765	3.3530	50.391	3.1218
1805	1 56 12.533	3.3540	2 52 6.001	3.1222
1810	29.305	3.3550	21.614	3.1227
1815	46.082	3.3560	37.228	3.1232
1820	1 57 2.865	3.3570	52.846	3.1237
1825	19.652	3.3580	2 53 8.465	3.1242
1830	36.445	3.3590	24.087	3.1246
1835	53.242	3.3600	39.712	3.1251
1840	1 58 10.045	3.3610	55.338	3.1256
1845	26.852	3.3620	2 54 10.968	3.1261
1850	43.665	3.3630	26.599	3.1266
1855	1 59 0.482	3.3640	42.233	3.1271
1860	17.305	3.3650	57.870	3.1275
1865	34.133	3.3661	2 55 13.509	3.1280
1870	50.966	3.3671	29.150	3.1285
1875	2 0 7.803	3.3681	44.794	3.1290
1880	24.646	3.3691	2 56 0.440	3.1295
1885	41.494	3.3701	16.088	3.1300
1890	58.347	3.3711	31.739	3.1304
1895	2 1 15.206	3.3721	47.393	3.1309
1900	32.069	3.3732	2 57 3.049	3.1314

Mean right ascensions, &c.—Continued.

Year.	α TAURI.		α AURIGÆ.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	4 21 36.293	3.4227	4 58 15.977	4.4004
1755	53.408	3.4232	37.981	4.4014
1760	4 22 10.526	3.4238	59.990	4.4023
1765	27.646	3.4243	4 59 22.004	4.4032
1770	44.769	3.4248	44.023	4.4042
1775	4 23 1.894	3.4254	5 0 6.046	4.4051
1780	19.022	3.4259	28.073	4.4060
1785	36.153	3.4265	50.106	4.4069
1790	53.286	3.4270	5 1 12.142	4.4078
1795	4 24 10.423	3.4275	34.184	4.4087
1800	27.562	3.4281	56.230	4.4096
1805	44.704	3.4286	5 2 18.280	4.4105
1810	4 25 1.848	3.4291	40.335	4.4114
1815	18.995	3.4296	5 3 2.394	4.4122
1820	36.144	3.4302	24.457	4.4131
1825	53.297	3.4307	46.525	4.4140
1830	4 26 10.452	3.4312	5 4 8.597	4.4148
1835	27.609	3.4318	30.673	4.4157
1840	44.769	3.4323	52.754	4.4165
1845	4 27 1.932	3.4328	5 5 14.839	4.4174
1850	19.097	3.4333	36.928	4.4182
1855	36.265	3.4339	59.022	4.4190
1860	53.436	3.4344	5 6 21.119	4.4199
1865	4 28 10.609	3.4349	43.221	4.4207
1870	27.785	3.4354	5 7 5.326	4.4215
1875	44.963	3.4359	27.436	4.4223
1880	4 29 2.144	3.4364	49.550	4.4232
1885	19.327	3.4370	5 8 11.668	4.4240
1890	36.514	3.4375	33.790	4.4248
1895	53.702	3.4380	55.916	4.4256
1900	4 30 10.894	3.4385	5 9 18.045	4.4264

Mean right ascensions, &c.—Continued.

Year.	β Orionis.		β Tauri.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	5 2 32.084	2.8757	5 10 30.599	3.7775
1755	46.463	2.8759	49.488	3.7779
1760	5 3 0.843	2.8761	5 11 8.378	3.7784
1765	15.224	2.8763	27.272	3.7788
1770	29.606	2.8765	46.167	3.7793
1775	43.989	2.8767	5 12 5.064	3.7797
1780	58.373	2.8769	23.964	3.7802
1785	5 4 12.758	2.8771	42.866	3.7806
1790	27.144	2.8773	5 13 1.770	3.7811
1795	41.531	2.8775	20.677	3.7815
1800	55.919	2.8777	39.585	3.7819
1805	5 5 10.309	2.8779	58.496	3.7824
1810	24.699	2.8781	5 14 17.409	3.7828
1815	39.090	2.8783	36.324	3.7832
1820	53.482	2.8785	55.241	3.7836
1825	5 6 7.875	2.8787	5 15 14.160	3.7840
1830	22.269	2.8789	33.081	3.7845
1835	36.665	2.8791	52.005	3.7849
1840	51.061	2.8793	5 16 10.930	3.7853
1845	5 7 5.458	2.8795	29.858	3.7857
1850	19.856	2.8797	48.787	3.7861
1855	34.256	2.8799	5 17 7.719	3.7865
1860	48.656	2.8801	26.652	3.7869
1865	5 8 3.057	2.8803	45.588	3.7873
1870	17.459	2.8805	5 18 4.526	3.7877
1875	31.862	2.8807	23.465	3.7881
1880	46.267	2.8809	42.407	3.7885
1885	5 9 0.672	2.8811	5 19 1.350	3.7889
1890	15.078	2.8813	20.296	3.7893
1895	29.485	2.8815	39.243	3.7897
1900	43.893	2.8817	58.193	3.7901

Mean right ascensions, &c.—Continued.

Year.	α ORIONIS.		α CANIS MAJORIS.*	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	5 41 38.680	3.2429	6 34 7.882	2.6446
1755	54.895	3.2431	21.105	2.6445
1760	5 42 11.111	3.2432	34.327	2.6445
1765	27.327	3.2434	47.550	2.6445
1770	43.545	3.2435	6 35 0.772	2.6444
1775	59.763	3.2437	13.994	2.6444
1780	5 43 15.982	3.2439	27.216	2.6444
1785	32.201	3.2440	40.438	2.6444
1790	48.422	3.2442	53.660	2.6443
1795	5 44 4.643	3.2443	6 36 6.881	2.6443
1800	20.865	3.2445	20.102	2.6443
1805	37.088	3.2446	33.324	2.6442
1810	53.311	3.2448	46.545	2.6442
1815	5 45 9.535	3.2449	59.766	2.6442
1820	25.760	3.2451	6 37 12.986	2.6441
1825	41.986	3.2452	26.207	2.6441
1830	58.212	3.2454	39.427	2.6441
1835	5 46 14.439	3.2455	52.647	2.6440
1840	30.667	3.2456	6 38 5.867	2.6440
1845	46.896	3.2458	19.087	2.6440
1850	5 47 3.125	3.2459	32.307	2.6439
1855	19.355	3.2461	45.527	2.6439
1860	35.586	3.2462	58.746	2.6439
1865	51.817	3.2464	6 39 11.965	2.6438
1870	5 48 8.050	3.2465	25.184	2.6438
1875	24.282	3.2466	38.403	2.6437
1880	40.516	3.2468	51.622	2.6437
1885	56.750	3.2469	6 40 4.840	2.6437
1890	5 49 12.985	3.2470	18.058	2.6436
1895	29.220	3.2472	31.276	2.6436
1900	45.457	3.2473	44.494	2.6436

* The corrections, P, on p. 69, are to be applied to these positions of α Canis Majoris.

Mean right ascensions, &c.—Continued.

Year.	α Canis Minoris.*		β Geminorum.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	7 26 12.027	3.1507	7 29 58.793	3.6964
1755	27.780	3.1505	7 30 17.274	3.6958
1760	43.532	3.1502	35.751	3.6952
1765	59.282	3.1499	54.225	3.6946
1770	7 27 15.031	3.1497	7 31 12.697	3.6940
1775	30.779	3.1494	31.165	3.6934
1780	46.525	3.1492	49.631	3.6928
1785	7 28 2.271	3.1489	7 32 8.093	3.6922
1790	18.015	3.1487	26.553	3.6916
1795	33.757	3.1484	45.009	3.6910
1800	49.499	3.1482	7 33 3.462	3.6904
1805	7 29 5.239	3.1479	21.913	3.6898
1810	20.978	3.1476	40.360	3.6892
1815	36.715	3.1474	58.804	3.6885
1820	52.452	3.1471	7 34 17.246	3.6879
1825	7 30 8.187	3.1469	35.684	3.6873
1830	23.920	3.1466	54.119	3.6867
1835	39.653	3.1463	7 35 12.551	3.6861
1840	55.384	3.1461	30.980	3.6855
1845	7 31 11.113	3.1458	49.405	3.6848
1850	26.842	3.1456	7 36 7.828	3.6842
1855	42.569	3.1453	26.248	3.6836
1860	58.295	3.1450	44.664	3.6829
1865	7 32 14.019	3.1448	7 37 3.077	3.6823
1870	29.742	3.1445	21.487	3.6817
1875	45.464	3.1442	39.894	3.6810
1880	7 33 1.185	3.1440	58.298	3.6804
1885	16.904	3.1437	7 38 16.699	3.6798
1890	32.622	3.1434	35.097	3.6791
1895	48.338	3.1432	53.491	3.6785
1900	7 34 4.053	3.1429	7 39 11.882	3.6778

* The corrections, P, on p. 71, are to be applied to these positions of α Canis Minoris.

Mean right ascensions, &c.—Continued.

Year.	α Hydræ.		α Leonis.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	9 15 17.906	2.9512	9 55 1.762	3.2148
1755	32.662	2.9512	17.835	3.2143
1760	47.418	2.9511	33.905	3.2138
1765	9 16 2.173	2.9510	49.973	3.2133
1770	16.928	2.9509	9 56 6.038	3.2128
1775	31.682	2.9508	22.100	3.2122
1780	46.436	2.9507	38.160	3.2117
1785	9 17 1.190	2.9507	54.217	3.2112
1790	15.943	2.9506	9 57 10.272	3.2107
1795	30.695	2.9505	26.324	3.2102
1800	45.448	2.9504	42.373	3.2096
1805	9 18 0.200	2.9503	58.420	3.2091
1810	14.951	2.9503	9 58 14.465	3.2086
1815	29.702	2.9502	30.507	3.2081
1820	44.453	2.9501	46.546	3.2076
1825	59.203	2.9500	9 59 2.582	3.2071
1830	9 19 13.953	2.9500	18.617	3.2066
1835	28.703	2.9499	34.648	3.2061
1840	43.452	2.9498	50.677	3.2056
1845	58.201	2.9497	10 0 6.704	3.2050
1850	9 20 12.950	2.9497	22.728	3.2045
1855	27.698	2.9496	38.749	3.2040
1860	42.446	2.9495	54.768	3.2035
1865	57.193	2.9495	10 1 10.784	3.2030
1870	9 21 11.940	2.9494	26.798	3.2025
1875	26.687	2.9493	42.810	3.2020
1880	41.433	2.9492	58.818	3.2015
1885	56.179	2.9492	10 2 14.825	3.2010
1890	9 22 10.925	2.9491	30.829	3.2005
1895	25.670	2.9490	46.830	3.2000
1900	40.416	2.9490	10 3 2.829	3.1995

Mean right ascensions, &c.—Continued.

Year.	β LEONIS.		α VIRGINIS.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	11 36 17.250	3.0745	13 12 3.448	3.1381
1755	32.622	3.0741	19.140	3.1386
1760	47.992	3.0737	34.834	3.1392
1765	11 37 3.360	3.0733	50.531	3.1397
1770	18.725	3.0729	13 13 6.231	3.1403
1775	34.089	3.0725	21.934	3.1408
1780	49.451	3.0722	37.639	3.1414
1785	11 38 4.811	3.0718	53.348	3.1419
1790	20.169	3.0714	13 14 9.059	3.1425
1795	35.525	3.0710	24.772	3.1430
1800	50.879	3.0706	40.489	3.1436
1805	11 39 6.231	3.0702	56.208	3.1441
1810	21.581	3.0699	13 15 11.930	3.1447
1815	36.929	3.0695	27.655	3.1452
1820	52.276	3.0691	43.382	3.1458
1825	11 40 7.620	3.0687	59.113	3.1464
1830	22.963	3.0683	13 16 14.846	3.1469
1835	38.304	3.0680	30.582	3.1475
1840	53.642	3.0676	46.321	3.1481
1845	11 41 8.980	3.0672	13 17 2.063	3.1486
1850	24.315	3.0669	17.807	3.1492
1855	39.648	3.0665	33.555	3.1498
1860	54.980	3.0661	49.305	3.1503
1865	11 42 10.309	3.0658	13 18 5.058	3.1509
1870	25.637	3.0654	20.814	3.1515
1875	40.963	3.0650	36.573	3.1520
1880	56.288	3.0647	52.334	3.1526
1885	11 43 11.610	3.0643	13 19 8.099	3.1532
1890	26.931	3.0640	23.866	3.1538
1895	42.250	3.0636	39.637	3.1544
1900	57.567	3.0633	55.410	3.1549

Mean right ascensions, &c.—Continued.

Year.	α BOOTIS.		α² LIBRÆ.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. *m.* *s.*	*s.*	*h.* *m.* *s.*	*s.*
1750	14 4 15.978	2.7320	14 37 5.726	3.2881
1755	29.638	2.7321	22.168	3.2888
1760	43.299	2.7322	38.614	3.2896
1765	56.960	2.7323	55.064	3.2904
1770	14 5 10.622	2.7324	14 38 11.518	3.2911
1775	24.284	2.7325	27.976	3.2919
1780	37.946	2.7326	44.437	3.2927
1785	51.609	2.7327	14 39 0.902	3.2934
1790	14 6 5.273	2.7328	17.371	3.2942
1795	18.937	2.7329	33.844	3.2950
1800	32.601	2.7330	50.321	3.2957
1805	46.266	2.7331	14 40 6.802	3.2965
1810	59.932	2.7332	23.286	3.2973
1815	14 7 13.598	2.7333	39.775	3.2980
1820	27.265	2.7334	56.267	3.2988
1825	40.932	2.7335	14 41 12.763	3.2996
1830	54.599	2.7336	29.263	3.3004
1835	14 8 8.267	2.7337	45.766	3.3011
1840	21.936	2.7338	14 42 2.274	3.3019
1845	35.605	2.7339	18.785	3.3026
1850	49.275	2.7340	35.301	3.3034
1855	14 9 2.945	2.7341	51.820	3.3042
1860	16.616	2.7342	14 43 8.343	3.3050
1865	30.288	2.7344	24.870	3.3058
1870	43.960	2.7345	41.401	3.3065
1875	57.633	2.7346	57.935	3.3073
1880	14 10 11.306	2.7347	14 44 14.474	3.3081
1885	24.980	2.7348	31.016	3.3089
1890	38.654	2.7350	47.562	3.3096
1895	52.329	2.7351	14 45 4.113	3.3104
1900	14 11 6.005	2.7352	20.667	3.3112

Mean right ascensions, &c.—Continued.

Year.	α CORONÆ.		α SERPENTIS.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. *m.* *s.*	*s.*	*h.* *m.* *s.*	*s.*
1750	15 24 6.587	2.5360	15 31 58.355	2.9431
1755	19.267	2.5361	15 32 13.071	2.9434
1760	31.948	2.5363	27.789	2.9437
1765	44.630	2.5364	42.508	2.9440
1770	57.312	2.5365	57.229	2.9443
1775	15 25 9.994	2.5366	15 33 11.952	2.9446
1780	22.678	2.5367	26.675	2.9449
1785	35.362	2.5368	41.401	2.9452
1790	48.046	2.5369	56.128	2.9455
1795	15 26 0.731	2.5370	15 34 10.856	2.9458
1800	13.416	2.5372	25.586	2.9462
1805	26.102	2.5373	40.318	2.9465
1810	38.789	2.5374	55.051	2.9468
1815	51.476	2.5375	15 35 9.785	2.9471
1820	15 27 4.164	2.5376	24.521	2.9474
1825	16.852	2.5377	39.259	2.9477
1830	29.541	2.5378	53.998	2.9480
1835	42.231	2.5380	15 36 8.739	2.9483
1840	54.921	2.5381	23.481	2.9486
1845	15 28 7.612	2.5382	38.225	2.9489
1850	20.303	2.5383	52.970	2.9492
1855	32.995	2.5384	15 37 7.717	2.9495
1860	45.687	2.5386	22.465	2.9498
1865	58.380	2.5387	37.215	2.9501
1870	15 29 11.074	2.5388	51.966	2.9504
1875	23.768	2.5389	15 38 6.719	2.9507
1880	36.463	2.5390	21.473	2.9510
1885	49.159	2.5392	36.229	2.9513
1890	15 30 1.855	2.5393	50.987	2.9516
1895	14.552	2.5394	15 39 5.746	2.9520
1900	27.249	2.5395	20.506	2.9523

Mean right ascensions, &c.—Continued.

Year.	α Scorpii.				α Herculis.			
	R. A.			Ann. Var.	R. A.			Ann. Var.
	h.	*m.*	*s.*	*s.*	*h.*	*m.*	*s.*	*s.*
1750	16	14	7.423	3.6487	17	3	15.554	2.7286
1755			25.669	3.6495			29.198	2.7288
1760			43.918	3.6503			42.842	2.7290
1765	16	15	2.172	3.6511			56.487	2.7291
1770			20.429	3.6519	17	4	10.133	2.7293
1775			38.690	3.6527			23.780	2.7295
1780			56.956	3.6534			37.428	2.7297
1785	16	16	15.225	3.6542			51.077	2.7299
1790			33.498	3.6550	17	5	4.727	2.7300
1795			51.775	3.6558			18.378	2.7302
1800	16	17	10.056	3.6566			32.029	2.7304
1805			28.341	3.6573			45.681	2.7306
1810			46.629	3.6581			59.335	2.7307
1815	16	18	4.922	3.6589	17	6	12.989	2.7309
1820			23.218	3.6596			26.644	2.7311
1825			41.518	3.6604			40.300	2.7313
1830			59.822	3.6612			53.956	2.7314
1835	16	19	18.130	3.6619	17	7	7.614	2.7316
1840			36.442	3.6627			21.273	2.7318
1845			54.757	3.6635			34.932	2.7320
1850	16	20	13.076	3.6642			48.592	2.7321
1855			31.399	3.6650	17	8	2.253	2.7323
1860			49.726	3.6657			15.915	2.7325
1865	16	21	8.057	3.6665			29.578	2.7327
1870			26.391	3.6673			43.242	2.7328
1875			44.730	3.6680			56.906	2.7330
1880	16	22	3.072	3.6688	17	9	10.572	2.7332
1885			21.417	3.6695			24.238	2.7333
1890			39.767	3.6703			37.905	2.7335
1895			58.120	3.6710			51.573	2.7337
1900	16	23	16.477	3.6718	17	10	5.242	2.7339

Mean right ascensions, &c.—Continued.

Year.	α Ophiuchi.		α Lyræ.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	17 23 20.430	2.7781	18 28 28.585	2.0298
1755	34.320	2.7783	38.734	2.0299
1760	48.212	2.7784	48.883	2.0299
1765	17 24 2.105	2.7786	59.033	2.0300
1770	15.998	2.7788	18 29 9.183	2.0300
1775	29.893	2.7790	19.334	2.0301
1780	43.788	2.7791	29.484	2.0301
1785	57.684	2.7793	39.635	2.0302
1790	17 25 11.581	2.7795	49.786	2.0303
1795	25.479	2.7797	59.938	2.0303
1800	39.377	2.7798	18 30 10.089	2.0304
1805	53.277	2.7800	20.241	2.0304
1810	17 26 7.177	2.7802	30.394	2.0305
1815	21.079	2.7803	40.546	2.0305
1820	34.981	2.7805	50.699	2.0306
1825	48.884	2.7807	18 31 0.852	2.0306
1830	17 27 2.788	2.7809	11.005	2.0307
1835	16.692	2.7810	21.159	2.0307
1840	30.598	2.7812	31.313	2.0308
1845	44.504	2.7814	41.467	2.0308
1850	58.411	2.7815	51.621	2.0309
1855	17 28 12.319	2.7817	18 32 1.776	2.0310
1860	26.228	2.7819	11.931	2.0310
1865	40.138	2.7820	22.086	2.0311
1870	54.049	2.7822	32.241	2.0311
1875	17 29 7.960	2.7824	42.397	2.0312
1880	21.872	2.7825	52.553	2.0312
1885	35.785	2.7827	18 33 2.709	2.0313
1890	49.699	2.7829	12.865	2.0313
1895	17 30 3.614	2.7830	23.022	2.0314
1900	17.530	2.7832	33.179	2.0314

Mean right ascensions, &c.—Continued.

Year.	γ Aquilæ.		α Aquilæ.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	19 34 22.396	2.8536	19 38 34.939	2.9302
1755	36.664	2.8536	49.590	2.9301
1760	50.932	2.8535	19 39 4.240	2.9300
1765	19 35 5.200	2.8535	18.890	2.9299
1770	19.467	2.8534	33.539	2.9298
1775	33.734	2.8534	48.188	2.9297
1780	48.001	2.8534	19 40 2.836	2.9296
1785	19 36 2.268	2.8533	17 484	2.9295
1790	16.534	2.8533	32.132	2.9295
1795	30.800	2.8532	46.779	2.9294
1800	45.066	2.8532	19 41 1.425	2.9293
1805	59.332	2.8531	16.071	2.9292
1810	19 37 13.597	2.8531	30.717	2.9291
1815	27.863	2.8530	45.362	2.9290
1820	42.128	2.8530	19 42 0.007	2.9289
1825	56.392	2.8529	14.652	2.9288
1830	19 38 10.657	2.8529	29.296	2.9287
1835	24.921	2.8528	43.939	2.9286
1840	39.185	2.8528	58.582	2.9286
1845	53.448	2.8527	19 43 13.225	2.9285
1850	19 39 7.712	2.8527	27.867	2.9284
1855	21.975	2.8526	42.508	2.9283
1860	36.238	2.8526	57.150	2.9282
1865	50.500	2.8525	19 44 11.790	2.9281
1870	19 40 4.763	2.8524	26.430	2.9280
1875	19.025	2.8524	41.070	2.9279
1880	33.287	2.8523	55.710	2.9278
1885	47.548	2.8523	19 45 10.349	2.9277
1890	19 41 1.810	2.8522	24.987	2.9276
1895	16.071	2.8522	39.625	2.9275
1900	30.331	2.8521	54.262	2.9274

Mean right ascensions, &c.—Continued.

Year.	β Aquilæ.		α² Capricorni.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	19 43 1.874	2.9490	20 4 9.754	3.3439
1755	16.619	2.9490	26.473	3.3435
1760	31.364	2.9489	43.189	3.3431
1765	46.108	2.9488	59.902	3.3426
1770	19 44 0.852	2.9488	20 5 16.614	3.3422
1775	15.595	2.9487	33.324	3.3418
1780	30.339	2.9486	50.032	3.3414
1785	45.082	2.9486	20 6 6.738	3.3410
1790	59.824	2.9485	23.442	3.3406
1795	19 45 14.567	2.9484	40.144	3.3402
1800	29.308	2.9483	56.844	3.3397
1805	44.050	2.9483	20 7 13.542	3.3393
1810	58.791	2.9482	30.237	3.3389
1815	19 46 13.532	2.9481	46.931	3.3385
1820	28.273	2.9481	20 8 3.622	3.3381
1825	43.013	2.9480	20.311	3.3376
1830	57.752	2.9479	36.999	3.3372
1835	19 47 12.492	2.9479	53.684	3.3368
1840	27.231	2.9478	20 9 10.367	3.3364
1845	41.970	2.9477	27.048	3.3360
1850	56.708	2.9476	43.726	3.3355
1855	19 48 11.446	2.9476	20 10 0.403	3.3351
1860	26.184	2.9475	17.078	3.3347
1865	40.921	2.9474	33.750	3.3343
1870	55.658	2.9473	50.421	3.3339
1875	19 49 10.394	2.9473	20 11 7.090	3.3334
1880	25.131	2.9472	23.756	3.3330
1885	39.866	2.9471	40.420	3.3326
1890	54.602	2.9471	57.082	3.3322
1895	19 50 9.337	2.9470	20 12 13.742	3.3317
1900	24.072	2.9469	30.400	3.3313

Mean right ascensions, &c.—Continued.

Year.	α CYGNI.		α AQUARII.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	20 32 54.937	2.0414	21 52 56.049	3.0888
1755	20 33 5.144	2.0415	21 53 11.492	3.0886
1760	15.352	2.0416	26.935	3.0884
1765	25.561	2.0417	42.376	3.0881
1770	35.770	2.0418	57.816	3.0879
1775	45.979	2.0419	21 54 13.255	3.0877
1780	56.189	2.0420	28.693	3.0875
1785	20 34 6.399	2.0421	44.130	3.0872
1790	16.610	2.0422	59.565	3.0870
1795	26.822	2.0423	21 55 15.000	3.0868
1800	37.034	2.0424	30.433	3.0866
1805	47.246	2.0425	45.865	3.0863
1810	57.459	2.0426	21 56 1.296	3.0861
1815	20 35 7.673	2.0428	16.726	3.0859
1820	17.887	2.0429	32.155	3.0857
1825	28.101	2.0430	47.583	3.0855
1830	38.316	2.0431	21 57 3.010	3.0852
1835	48.532	2.0432	18.435	3.0850
1840	58.748	2.0433	33.860	3.0848
1845	20 36 8.965	2.0434	49.283	3.0846
1850	19.182	2.0435	21 58 4.706	3.0844
1855	29.399	2.0436	20.127	3.0842
1860	39.618	2.0437	35.548	3.0840
1865	49.836	2.0438	50.967	3.0838
1870	20 37 0.056	2.0439	21 59 6.385	3.0835
1875	10.276	2.0440	21.802	3.0833
1880	20.496	2.0441	37.218	3.0831
1885	30.717	2.0443	52.633	3.0829
1890	40.939	2.0444	22 0 8.048	3.0827
1895	51.161	2.0445	23.461	3.0825
1900	20 38 1.383	2.0446	38.873	3.0823

Mean right ascensions, &c.—Continued.

Year.	α Piscium Australis.		α Pegasi.	
	R. A.	Ann. Var.	R. A.	Ann. Var.
	h. m. s.	*s.*	*h. m. s.*	*s.*
1750	22 43 46.710	3.3551	22 52 19.531	2.9775
1755	22 44 3.483	3.3540	34.419	2.9778
1760	20.250	3.3529	49.309	2.9780
1765	37.012	3.3518	22 53 4.199	2.9783
1770	53.768	3.3507	19.091	2.9785
1775	22 45 10.518	3.3496	33.985	2.9788
1780	27.264	3.3485	48.879	2.9790
1785	44.003	3.3474	22 54 3.775	2.9793
1790	22 46 0.737	3.3463	18.672	2.9795
1795	17.466	3.3452	33.570	2.9798
1800	34.189	3.3441	48.470	2.9801
1805	50.907	3.3430	22 55 3.371	2.9803
1810	22 47 7.619	3.3419	18.273	2.9806
1815	24.326	3.3408	33.177	2.9809
1820	41.028	3.3397	48.082	2.9811
1825	57.724	3.3387	22 56 2.988	2.9814
1830	22 48 14.414	3.3376	17.896	2.9817
1835	31.100	3.3365	32.805	2.9819
1840	47.779	3.3354	47.715	2.9822
1845	22 49 4.454	3.3343	22 57 2.627	2.9825
1850	21.123	3.3333	17.540	2.9828
1855	37.786	3.3322	32.455	2.9830
1860	54.445	3.3311	47.370	2.9833
1865	22 50 11.097	3.3300	22 58 2.288	2.9836
1870	27.745	3.3290	17.206	2.9839
1875	44.387	3.3279	32.126	2.9841
1880	22 51 1.024	3.3268	47.048	2.9844
1885	17.656	3.3258	22 59 1.971	2.9847
1890	34.282	3.3247	16.895	2.9850
1895	50.903	3.3236	31.821	2.9853
1900	22 52 7.518	3.3226	46.748	2.9856

Periodic terms to be applied to the right ascensions of Sirius and Procyon.

α Canis Majoris.				
Year.	Year.	Year.	P	
			s.	
1750.6	1800.0	1849.4	+.026	−20
1751.6	1801.0	1850.4	+.006	−20
1752.6	1802.0	1851.4	−.014	−17
1753.6	1803.0	1852.4	−.031	−16
1754.6	1804.0	1853.4	−.047	−15
1755.6	1805.0	1854.4	−.062	−14
1756.6	1806.0	1855.4	−.076	−12
1757.6	1807.0	1856.4	−.088	−11
1758.6	1808.0	1857.4	−.099	−10
1759.6	1809.0	1858.4	−.109	−9
1760.6	1810.0	1859.4	−.118	−8
1761.6	1811.0	1860.4	−.126	−6
1762.6	1812.0	1861.4	−.132	−6
1763.6	1813.0	1862.4	−.138	−5
1764.6	1814.0	1863.4	−.143	−4
1765.6	1815.0	1864.4	−.147	−2
1766.6	1816.0	1865.4	−.149	−2
1767.6	1817.0	1866.4	−.151	−1
1768.6	1818.0	1867.4	−.152	0
1769.6	1819.0	1868.4	−.152	+1
1770.6	1820.0	1869.4	−.151	+2
1771.6	1821.0	1870.4	−.149	+3
1772.6	1822.0	1871.4	−.146	+4
1773.6	1823.0	1872.4	−.142	+5
1774.6	1824.0	1873.4	−.137	+6
1775.6	1825.0	1874.4	−.131	+7
1776.6	1826.0	1875.4	−.124	+7
1777.6	1827.0	1876.4	−.117	+9
1778.6	1828.0	1877.4	−.108	+10
1779.6	1829.0	1878.4	−.098	+12
1780.6	1830.0	1879.4	−.086	+12
1781.6	1831.0	1880.4	−.074	+13